Time-Series Constraints: Improvements and Application in CP and MIP Contexts

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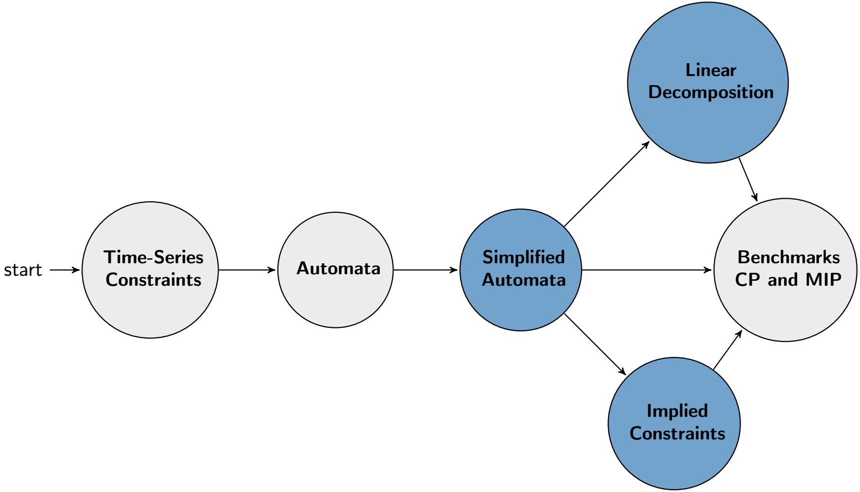
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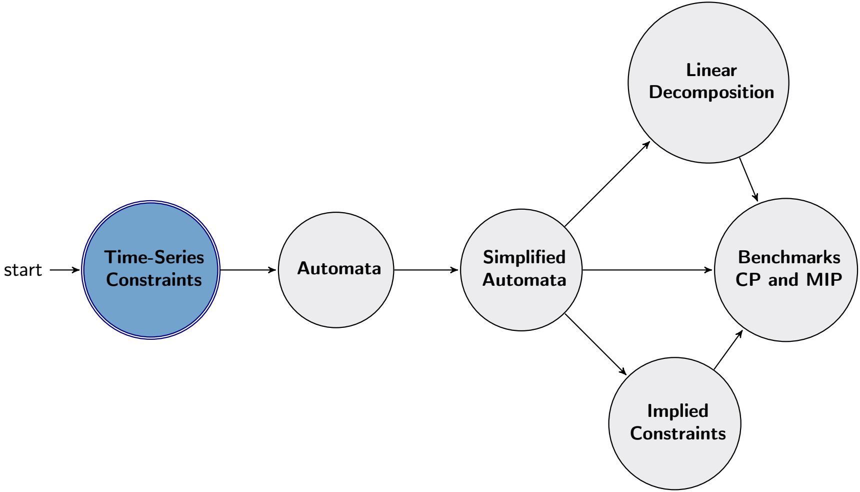
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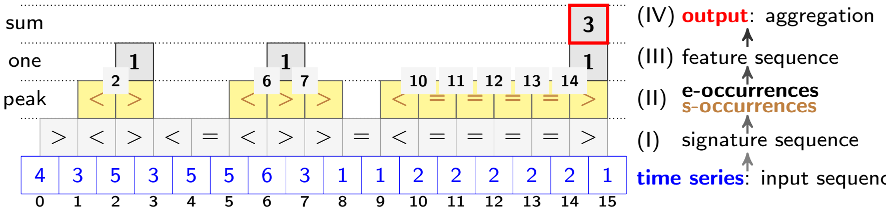
Time-series constraint

A time-series constraint[1] $g\_f\_\sigma(\langle X_1, \ldots, X_n \rangle, M)$ where every X_i is over $D_i \subset \mathbb{Z}$ is specified by

- A pattern, a regular expression over the alphabet $\{<, =, >\}$, e.g. Peak = '<(<|=)*(>|=)*>'.
  Currently 22 patterns in the framework
- A feature, a function over a subseries, e.g. one.
  Currently 5 features in the framework
- An aggregator, a function over a feature sequence, e.g. Sum.
  Currently 3 aggregators in the framework

[1] Beldiceanu, N., Carlsson, M., Douence, R., Simonis, H.: Using finite transducers for describing and synthesising structural time-series constraints. Constraints 21(1), 22-40 (January 2016): summary on p. 723 of LNCS 9255, Springer, 2015

NbPeak



Example

NbPeak$(\langle 4, 3, 5, 3, 5, 5, 6, 3, 1, 1, 2, 2, 2, 2, 2, 1\rangle, 3)$ holds !

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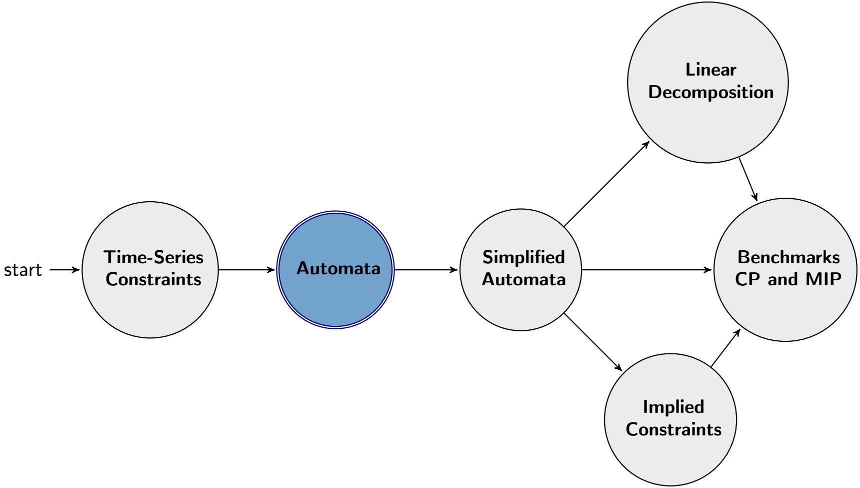
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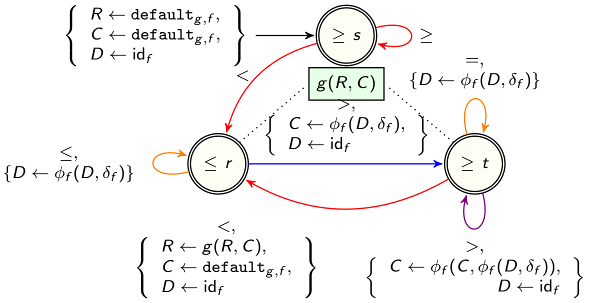
Automata for time-series constraints

Every time-series constraint can be encoded as an automaton with three accumulators: $D(potential)$, $C(current)$, $R(aggregation)$



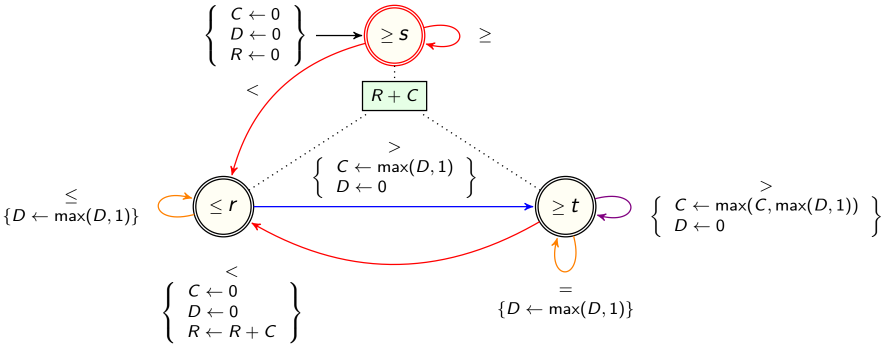
Automaton for the g_f_peak constraints.

Feature f	id_f	$\min_f$	$\max_f$	ϕ_f	δ_f^i
one	1	1	1	max	0

Aggregator g	$\mathrm{default}_{g,f}$
Sum	0

Automaton instantiation

When f is one and g is $\mathtt{Sum}$ the automaton becomes



Obviously, this automaton can be simplified

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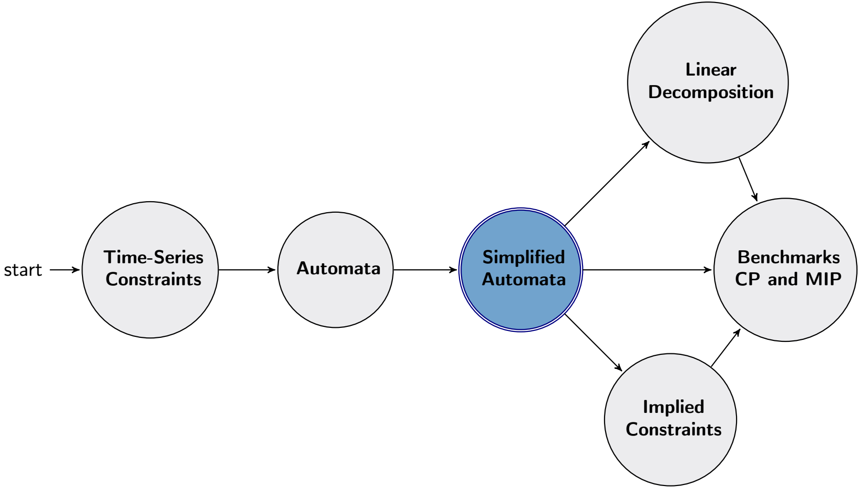
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Automata simplifications

Goal

- **Reduce** the number of accumulators and aggregate as **early** as possible
- Simplify the automata at the stage of their synthesis

Three simplification types

- Simplifications coming from the properties of patterns, ex.: aggregate-once
- Simplifications coming from the properties of the feature/aggregator pairs, ex.: immediate-aggregation
- Removing the never used accumulators.

“Aggregate-once” simplification

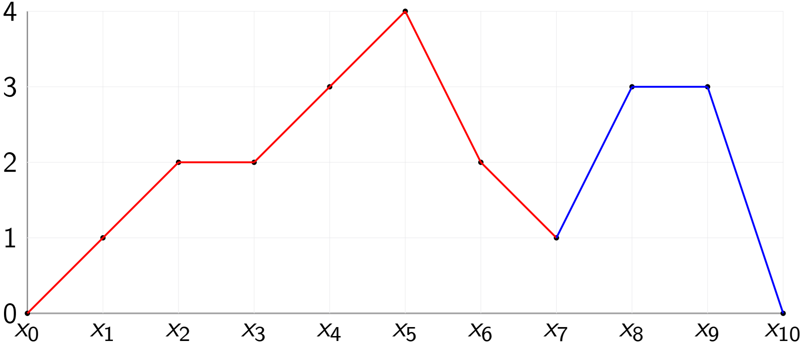
What is the “Aggregate-once” simplification ?

It allows to compute the feature value of a curent pattern occurrence only once and, possibly, earlier than the end of a pattern occurrence.

When is the simplification applicable ?

There must exist a transition on which the value of the feature from the current pattern occurrence is known.

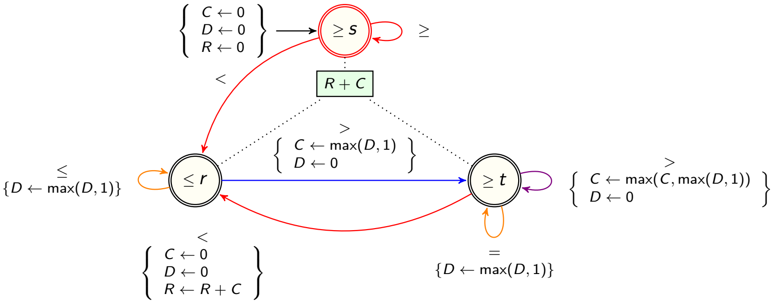
Example: counting number of peaks

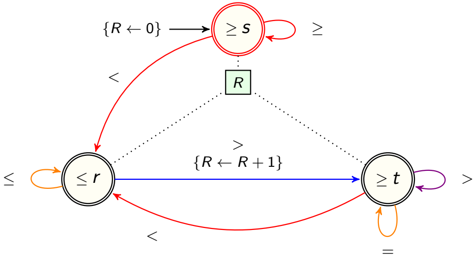


$s_0 =$‘<’ $s_1 =$‘<’ $s_2 =$‘=’ $s_3 =$‘<’ $s_4 =$‘<’ $s_5 =$‘>’ $s_6 =$‘>’ $s_7 =$‘<’ $s_8 =$‘=’ $s_9 =$‘>’

1. First peak is detected upon consuming s_5
2. Second peak is detected upon consuming s_9

Two automata for nb_peak





Percentage of simplified constraints

Simplification	Percentage
aggregate once	28.9 %
immediate aggreg.	45.9 %
other properties	11.6 %
unchanged automata	13.6 %

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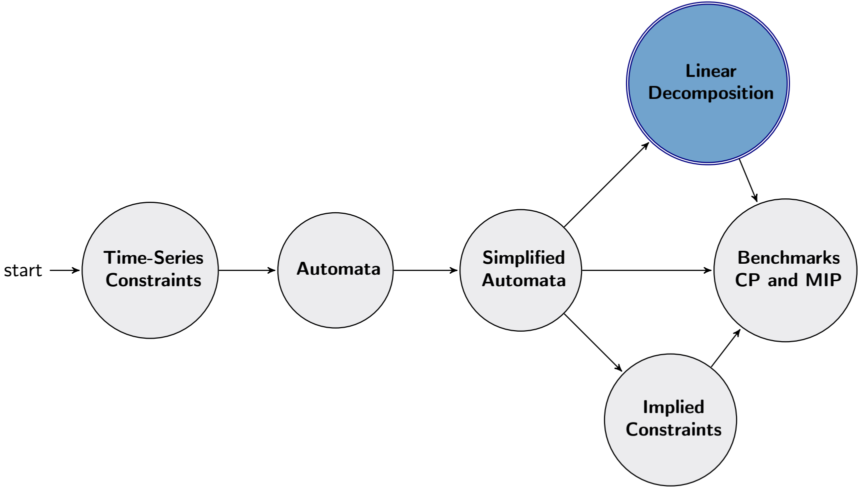

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Input

Input

- Time-series variables X_i with i in $[0, n-1]$ over their domains $[a_i, b_i]$
- An automaton with accumulators for a time-series constraint with
  - a set of states Q;
  - an input alphabet Σ;
  - an m-tuple of integer accumulators with their initial values $I = \langle I_1, \ldots, I_m \rangle$;
  - a transition function $\delta : Q \times Z^m \times \Sigma \rightarrow Q \times Z^m$.

Goal

Goal

A way to generate a model for an automaton with linear or linearisable accumulator updates, for example containing min and max.

Linear decomposition of automata without accumulators

Côté, M.C., Gendron, B., Rousseau, L.M.: Modeling the regular constraint with integer programming. In: CPAIOR 2007. LNCS, vol. 4510, pp. 29–43. Springer (2007)

Signature constraint

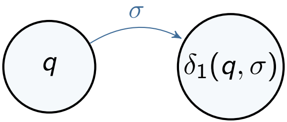
Introduced variables: S_i over Σ with $i \in [0, n-2]$.

What do the values of S_i mean ?

$$S_i = \text{`}{>}\text{'} \Leftrightarrow X_i > X_{i+1}, \forall i \in [0, n-2]$$
$$S_i = \text{`}{=}\text{'} \Leftrightarrow X_i = X_{i+1}, \forall i \in [0, n-2]$$
$$S_i = \text{`}{<}\text{'} \Leftrightarrow X_i < X_{i+1}, \forall i \in [0, n-2]$$

Transition function constraints

Introduced variables: Q_i over Q with $i \in [0, n-1]$; T_i over $Q \times \Sigma$ with $i \in [0, n-2]$



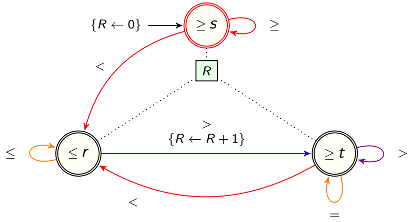
Each transition constraint has a form:

$Q_i = q \wedge S_i = \sigma \Leftrightarrow Q_{i+1} = \delta_1(q, \sigma) \wedge T_i = \langle q, \sigma \rangle$,
$\forall i \in [0, n-2],\ \forall q \in Q,\ \forall \sigma \in \Sigma$

Initial state is fixed

$Q_0 = q_0$

Accumulator updates



Accumulator updates

R_i over $[a, b]$ with i in $[0, n-1]$; T_i over $Q \times \Sigma$ with i in $[0, n-2]$.

- $R_0 = 0$
- $T_i = \langle r, > \rangle \Rightarrow \mathbf{R_{i+1} = R_i + 1}, \forall i \in [0, n-2]$
- $T_i = \langle q, \sigma \rangle \Rightarrow \mathbf{R_{i+1} = R_i}, \forall i \in [0, n-2], \forall \langle q, \sigma \rangle \in (Q \times \Sigma) \setminus \langle r, > \rangle$
- $M = R_{n-1}$

New variables for the linear model

New variables

- Q_i is replaced by 0-1 variables Q_i^q for all q in Q.
  $Q_i^q = 1 \Leftrightarrow Q_i = q$
- New constraint: $\sum_{q \in Q} Q_i^q = 1, \forall i \in [0, \ldots, n-1]$
- The same procedure for T_i and S_i wrt their domains
- X_i and R_i remain integer variables!

- Every constraint of the logical model is made linear by applying some standard techniques
- The linear model has $O(n)$ variables and $O(n)$ constraints

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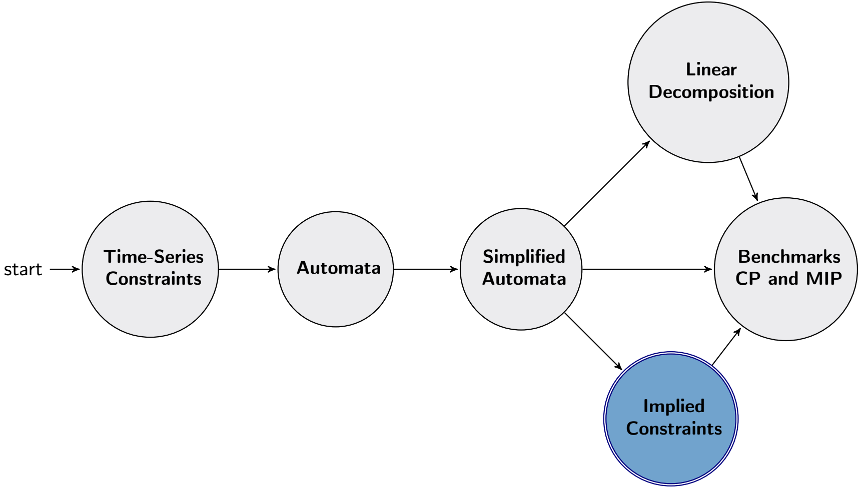
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Implied constraints

Implied constraints[2] improves propagation for constraints encoded via automata with at least one accumulator

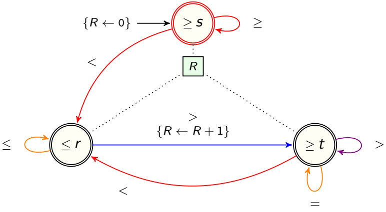
- The implied constraints are generated offline
- The implied constraints are of the form:

$$\alpha_1 y_1 + \cdots + \alpha_k y_k + \beta \geq 0$$

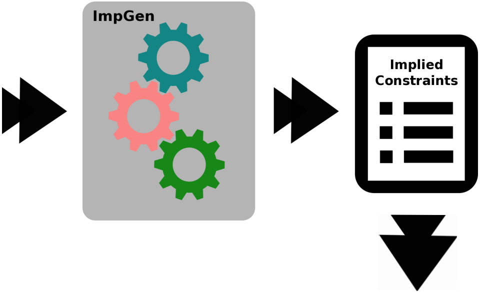
where the y_i are the accumulators of $\mathcal{A}(C,\ D,\ R)$ and the weights α_i and β are to be found

- Theoretically supported by Farkas' Lemma

[2] Francisco Rodríguez, M.A., Flener, P., Pearson, J.: Implied constraints for automaton constraints. In: GCAI 2015. EasyChair Epic Series in Computing, vol. 36, pp. 113–126. EasyChair (2015)



Options



$$R_{i+2} \leq R_i + 1$$

Improvements

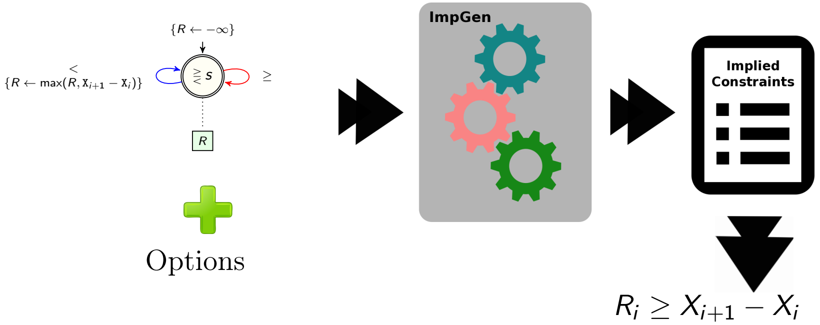
The first version of `ImpGen`

- Only linear accumulator updates
- Manual selection

Improvements of the new version

- Can handle max and min in accumulators updates
- Automatic selection by ranking

Improvements of `ImpGen`: example



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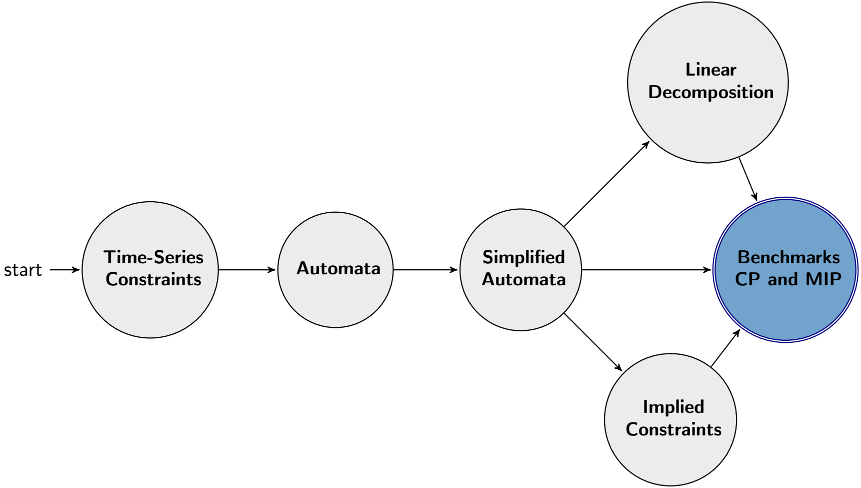
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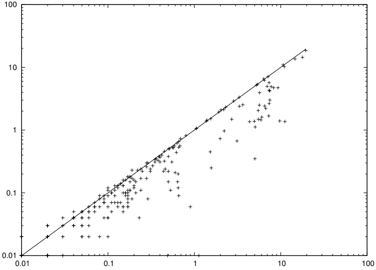
Benchmark CP

Goal

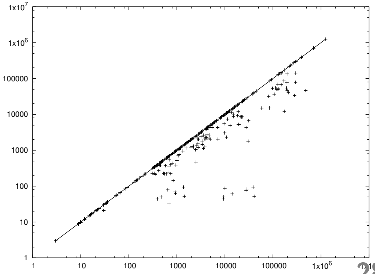
compare original and simplified automata

- For every time-series constraint maximise the result
- Time series of length 15 over $[1, 3]$
- Timeout of 100 seconds

(a) Runtime

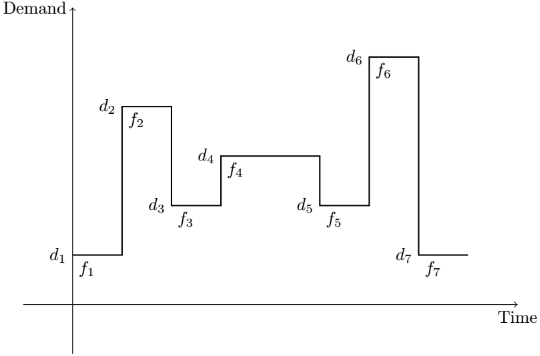


(b) Backtracks



Staff scheduling application

Figure 1: Typical Benchmark Demand Profile



- Satisfy the demand;
- Take into account business rules
- Respect union's rules
- Minimise the costs

Results for staff scheduling application

- P characterises complexity of the problem
- Consider $P \in \{10, 15, 20, 25, 30, 35, 40\}$
- 100 instances for every value of P

optimality gap					
	cp		mip		
p	avg	max	avg	max	opt
20	3.42	9.67	2.28	18.77	27/100
30	3.20	8.02	2.04	6.34	26/100
40	3.51	17.32	1.97	10.47	18/100

- In average MIP is always better
- The maximal gap sometimes is smaller for CP
- MIP can solve to optimality just few instances

Conclusion

Contributions of the paper

- A linear decomposition for time-series constraints with $O(n)$ variables and $O(n)$ constraints
- Simplified automata for time-series constraints
- New version of the generator of linear implied constraints which handles accumulator updates with min, max
- Benchmarks in the contexts of CP and MIP

Thank you for your attention!
Questions?