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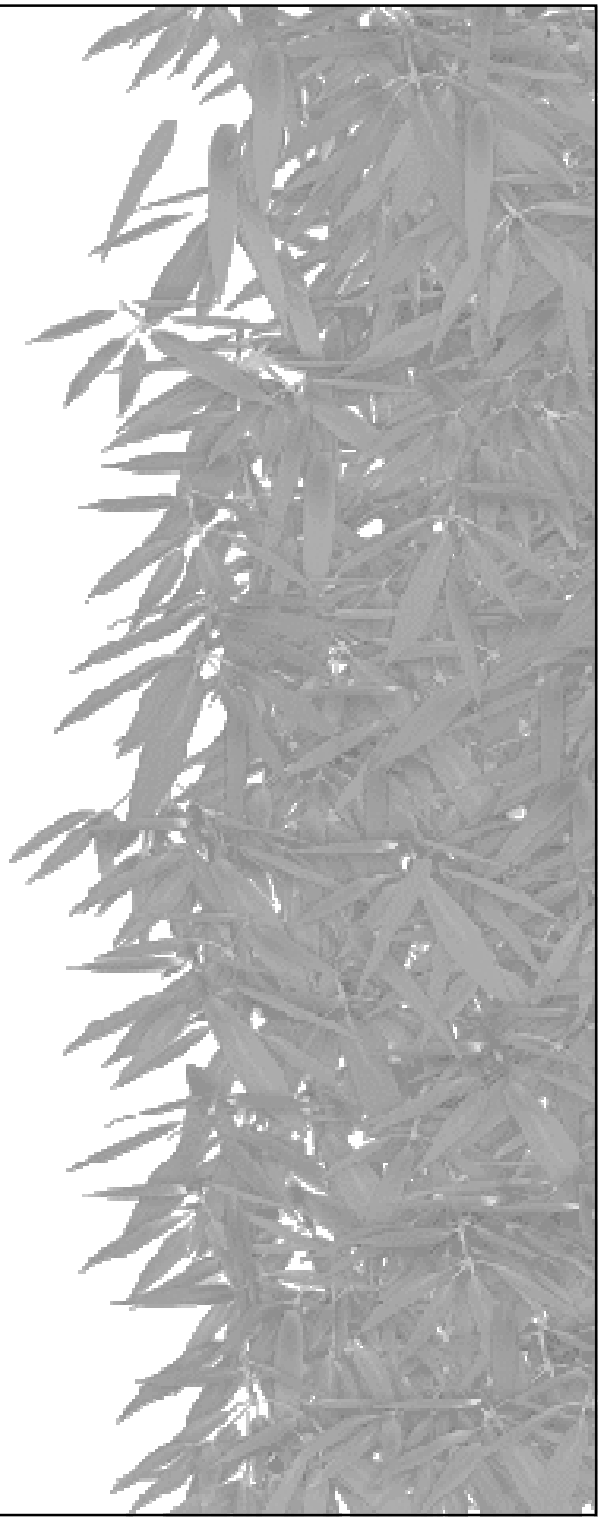

PartialSymmetry Breaking

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Barbara Smith

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Motivations for PSB

- Curiosity
  - What happens when we don't break all the symmetry that exists in a CSP?
  - Is it ever quicker to break less symmetry and perform some redundant search?
- Highly symmetric problems

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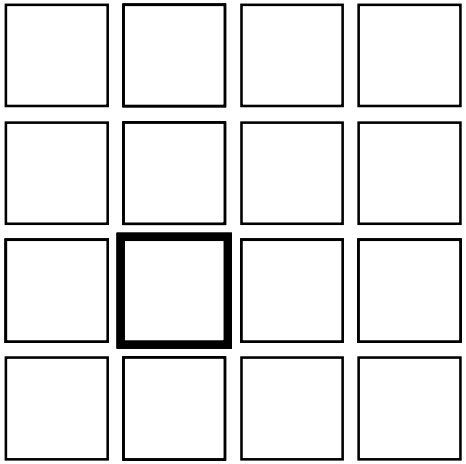
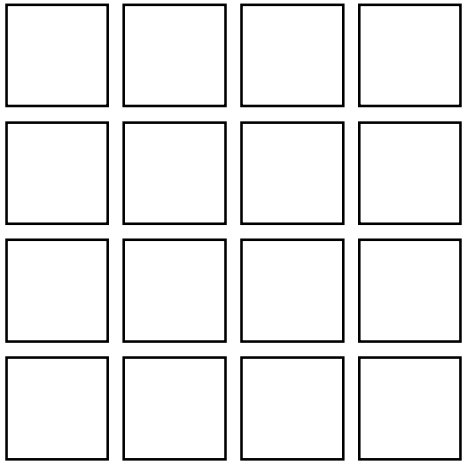
Alien Tiles Problem



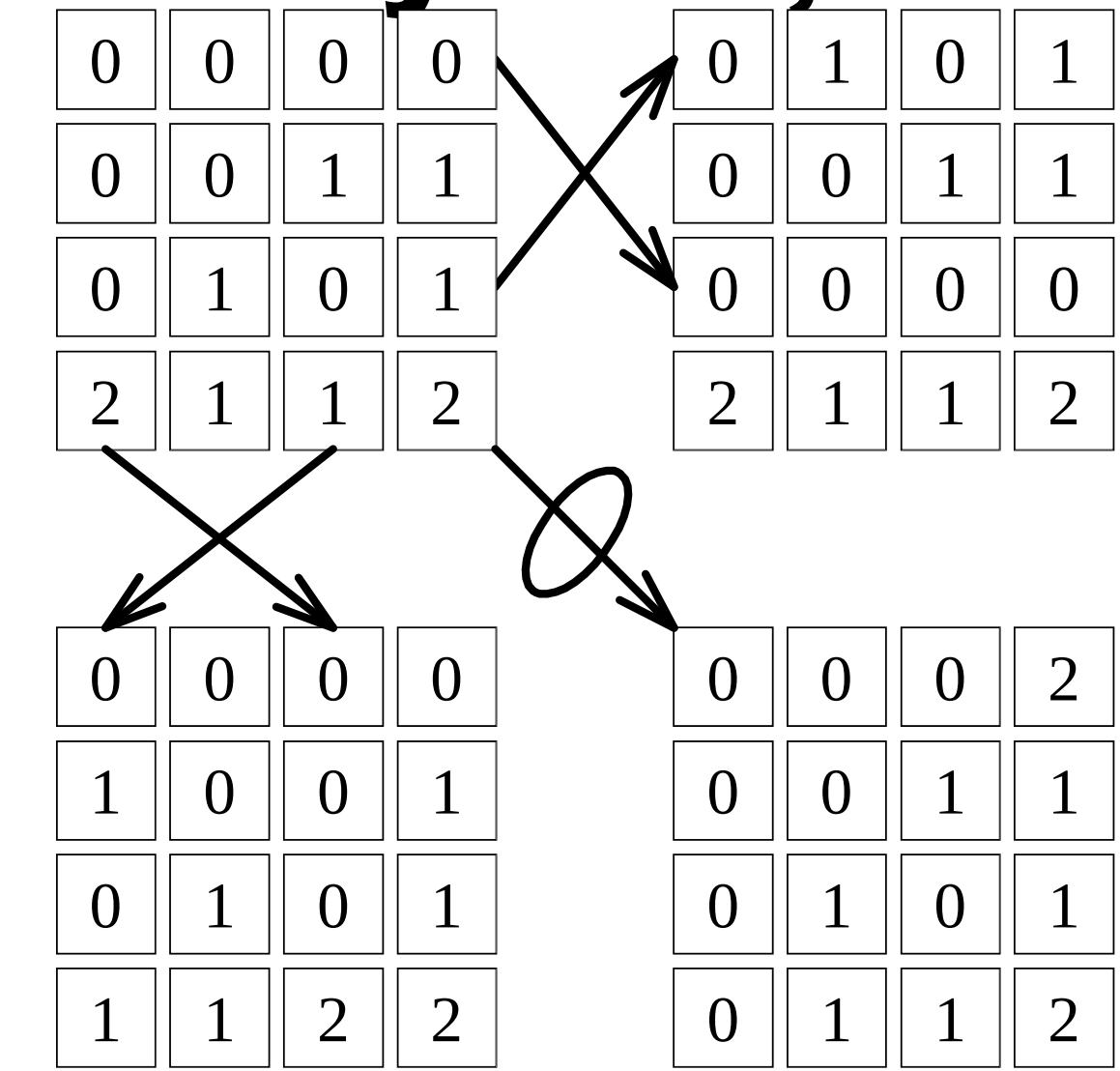
Initial State

Solution

0	0	0	0
0	0	1	1
0	1	0	1
2	1	1	2



Goal State


0 0 0 0
0 0 1 1
0 1 0 1
2 1 1 2
0 1 0 1
0 0 1 1
0 0 0 0
2 1 1 2
0 0 0 0
1 0 0 1
0 1 0 1
1 1 2 2
0 0 0 2
0 0 1 1
0 1 0 1
0 1 1 2

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PSBExperiment

- Solve Alien Tiles problem
  - Record cpu-time
- Solve again with SBDS
  - Pick a symmetry at random
- Repeat with a random subset of symmetries one larger than last time until the problem is solved with all 1,151 symmetries

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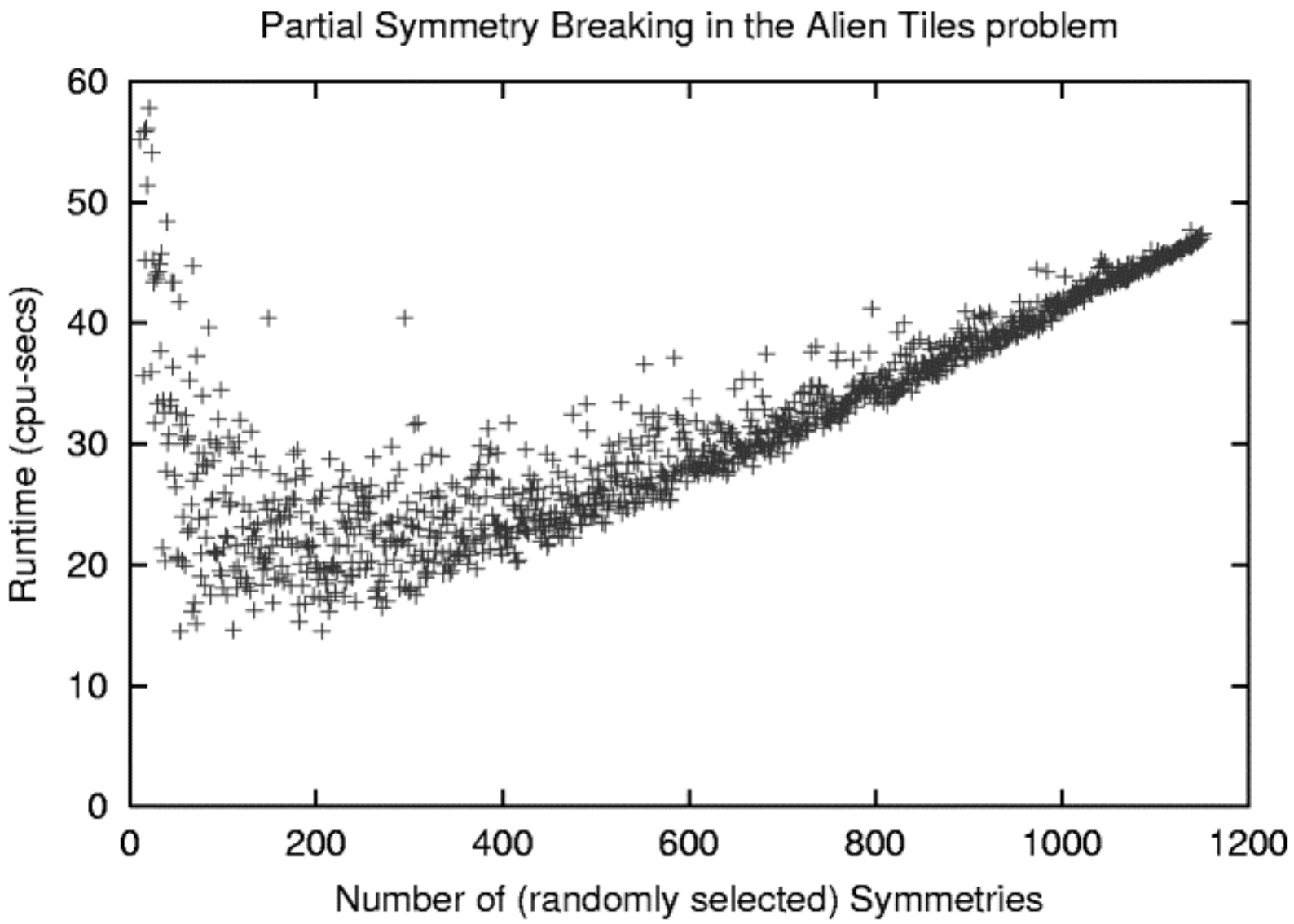
InitialResults



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NewMotivations

- Why is one subset of symmetries better than another?
- Can we break all symmetry with just a subset of symmetries?
- Is there a way to find the trough of the curve?

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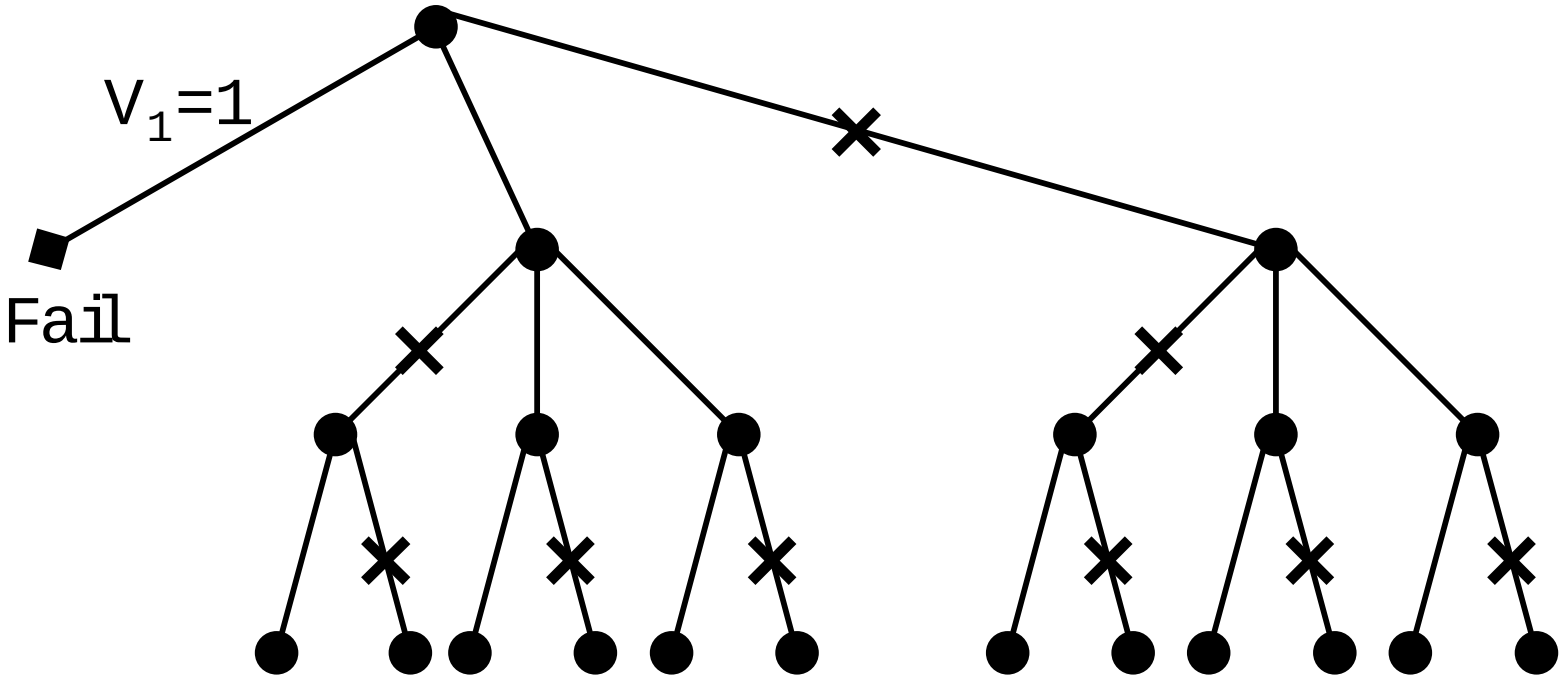
Finding the good subsets



$f(V_1=1) = (V_3=2)$

$g(V_1=1) = (V_1=3)$

PSB_Algorithm(f, g, h) = {g, h, f}

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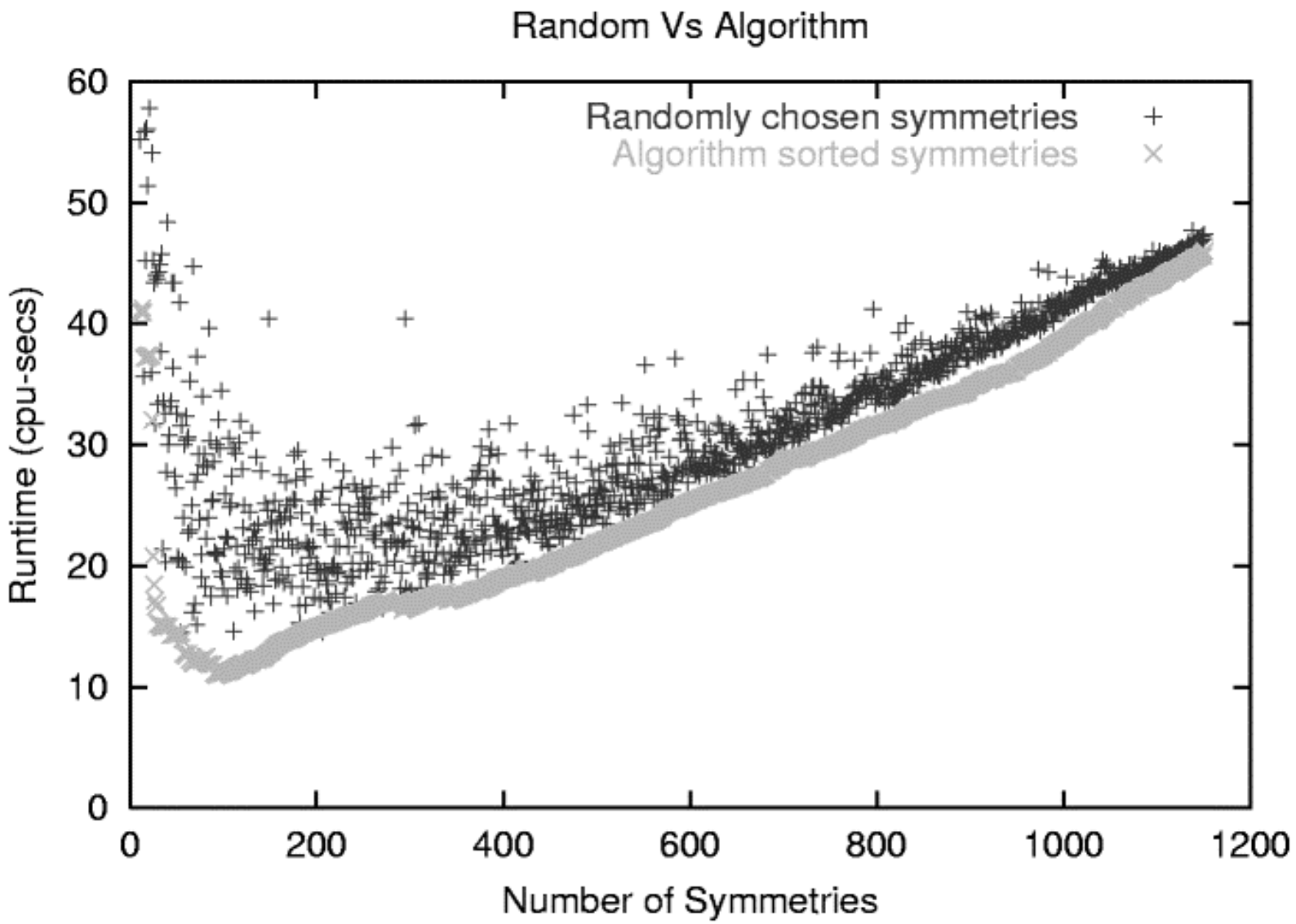
More results: Alien Tiles



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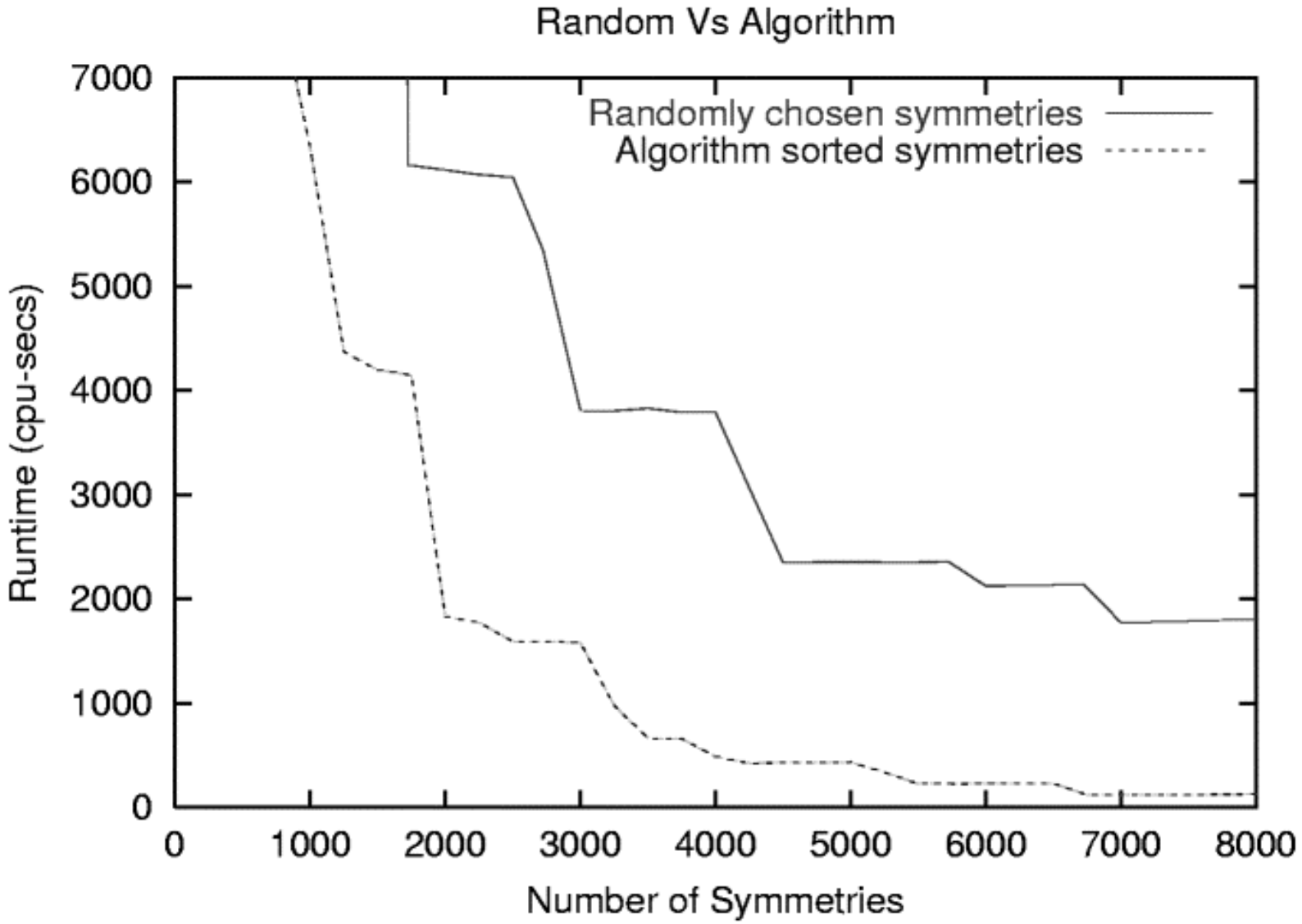
Golfer's Problem



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Breaking all Symmetry

- Cats and Dogs
- Interchangeable cats (3! = 6), interchangeable dogs (5! = 120) and symmetries of a square (8)
  - $6 \times 120 \times 8 = 5{,}760$
- Break all symmetry with 41 functions

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GeneralityofPSB

- Group theory / Dominance check
- SBDD
- Dynamic PSB

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Motivations Revisited

- Why is one subset of symmetries better than another? ✓
- Can we break all symmetry with just a subset of symmetries? ✓
- Is there a way to find the trough of the curve? ✗

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Futurework

- First solution
- Find optimum number of symmetries
- Combining PSB with other research
  - Nu-SBDS

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Conclusion

- Thanks to APES research group, CP2002 program committee, reviewers, supervisor, co-author and St Andrews research group…
- Nu-SBDS (and my CV) can be found at http://www.dcs.st-and.ac.uk/~iain/ :o)

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AlienTiles

- Each board state belongs to an equivalence class. For any given goal state, it may be possible to find a full assignment that maps the initial state, to a state in the same equivalence class as the goal state, that takes less clicks than the previous solution. If so, the full assignment becomes the new solution.

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Golfer's Problem

- *p* players must split up into *g* groups, to play golf with each other. The golfers must do this for *w* weeks. Any two golfers can play with each other at most once.
- The example used in this presentation was *12* golfers, *4* groups (of 3) played for *2* weeks.

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Cats and Dogs

- *3* cats and *5* dogs must be placed on a 5×5 chess board such that no dog can attack a cat in a queen's move.

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Static Heuristics

- Heuristics not essential for optimization problems or finding all solutions
- Dynamic PSB will be able to find *next* variable
- For highly symmetric problems, benefit of PSB will far outweigh benefit of dynamic heuristics