# Motivation

- The effect of symmetry breaking is often evaluated empirically

- It is possible to evaluate it on the basis of the underlying symmetry structure ?

- Can the group theory help us in this task ?

- We start with a VERY simple case and try to generalize

# Symmetry in matrix models

- Row and column symmetries:
  - *two matrices are symmetric if one can be obtained from the other by any row and/or column permutation*

- Many ways to remove these symmetries
  - *row sum - column sum*
  - *row lex - column lex*
  - *row sum - column lex*

- Which method is stronger ?

# Group theory can help

- A group is a tuple G = (S,Op) where S is a set and Op a closed binary operation over S.

- Properties:
  - *Op associative*
  - *S contains a neutral element I*
  - *each element in S has an inverse*

- Group generators are movement which enable (if combined through Op) to obtain all the group elements.

# Group theory can help

- A group has an associated graph:
  - *nodes are group elements*
  - *arcs are generators*
  - *paths are compositions of generators*

- Each node can be labelled through the composition of generators applied to reach it.
  - *Is the choice of generators important ?*

- Consider small matrices 3 x 3

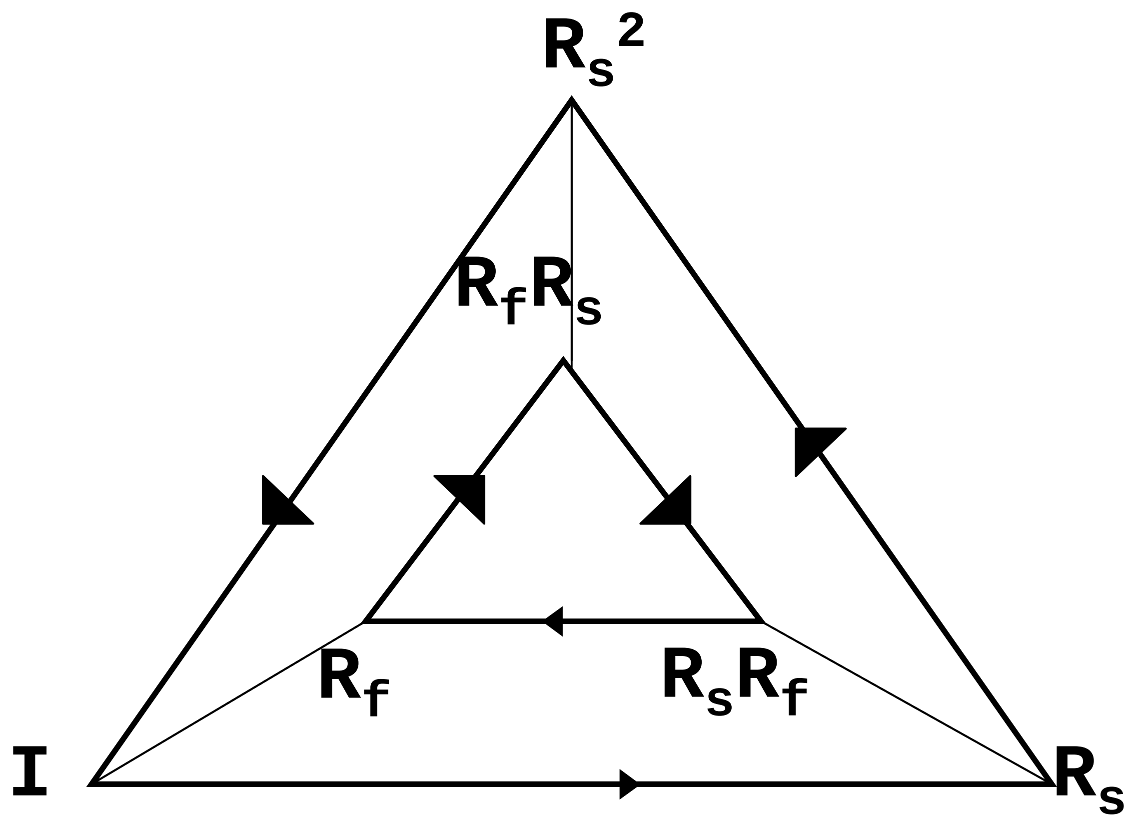
# Row permutation (3x3 matr.)

- Non commutative group with 6 elements
  - the graph has 6 nodes

- Two generators:
  - *flip of the first 2 rows* ($R_f$) *period 2*
  - *shift of the rows* ($R_s$) *period 3*

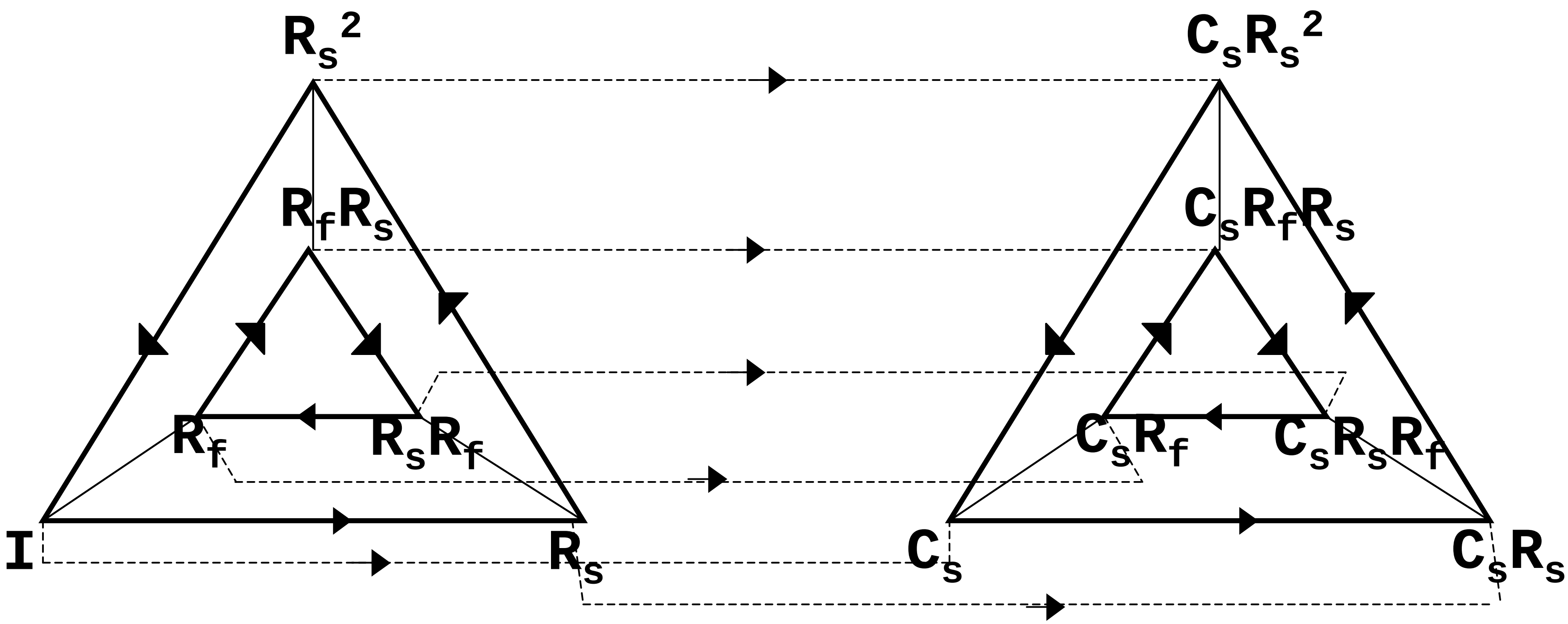
  the graph has 2 incoming and 2 outcoming arcs from each node.

# Row permutation (3x3 matr.)

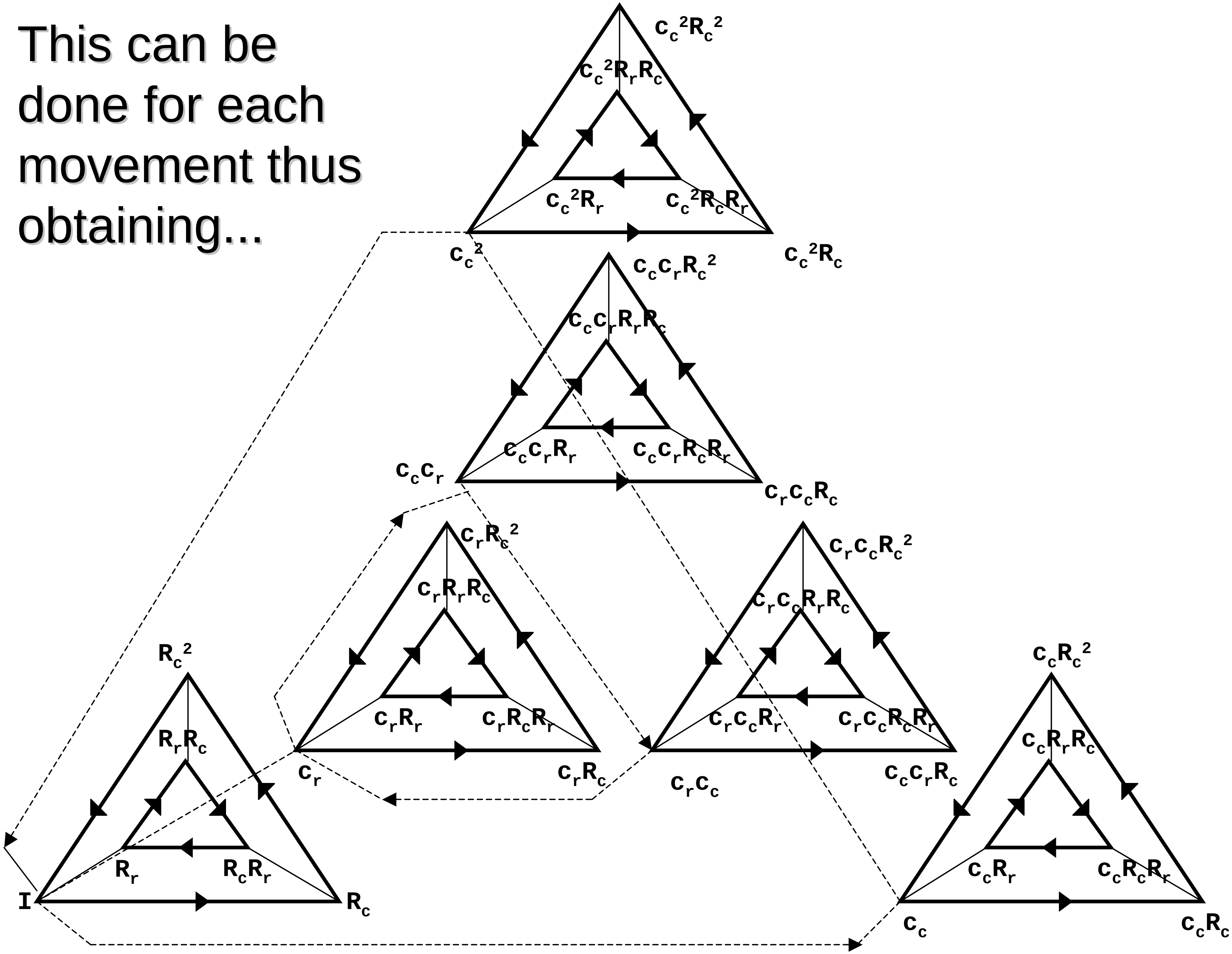


# Adding column permutation

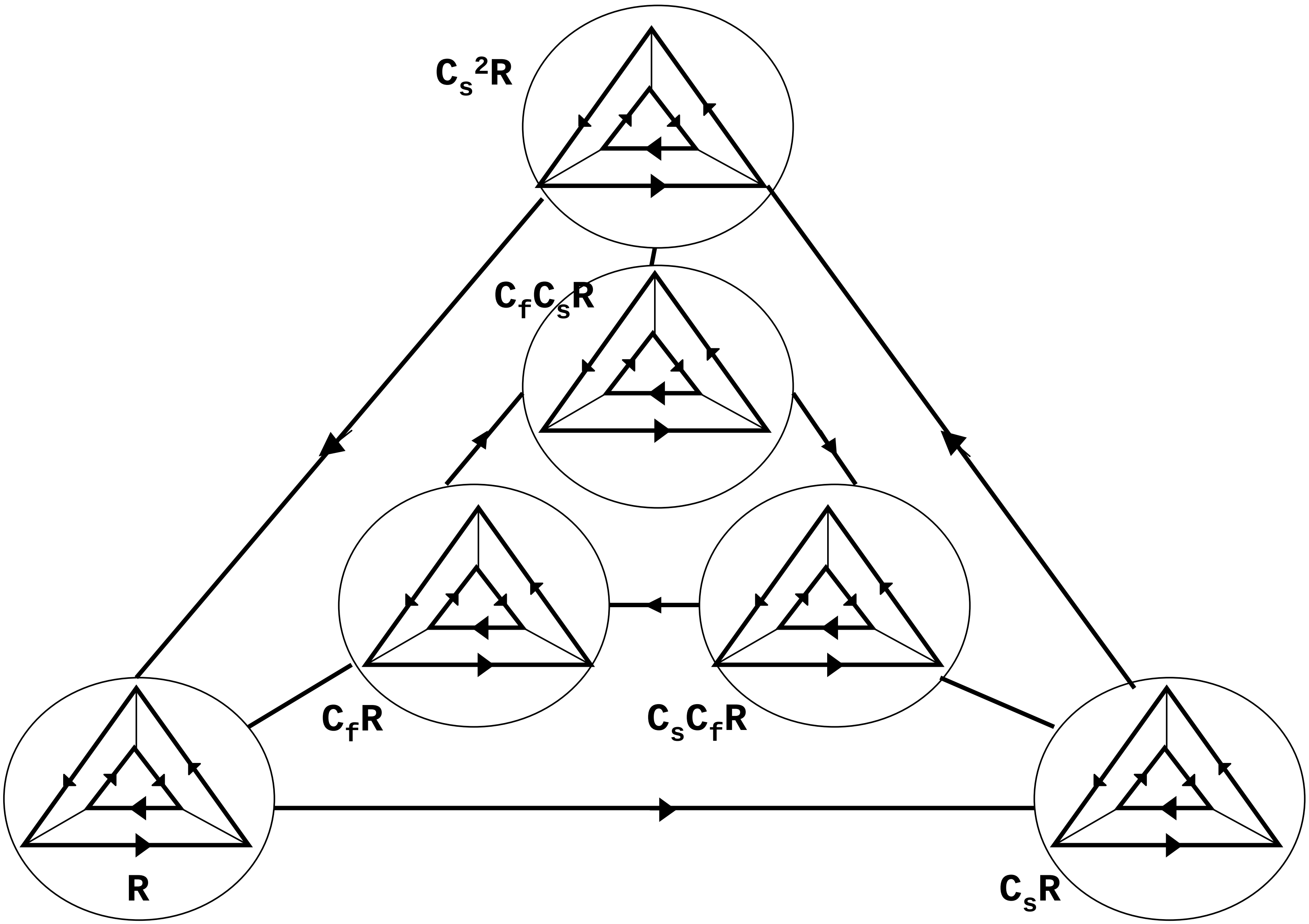
- In each vertex of this graph we can apply a column movement (Cf or Cs) and obtain the corresponding graph



This can be done for each movement thus obtaining...



# Complete graph



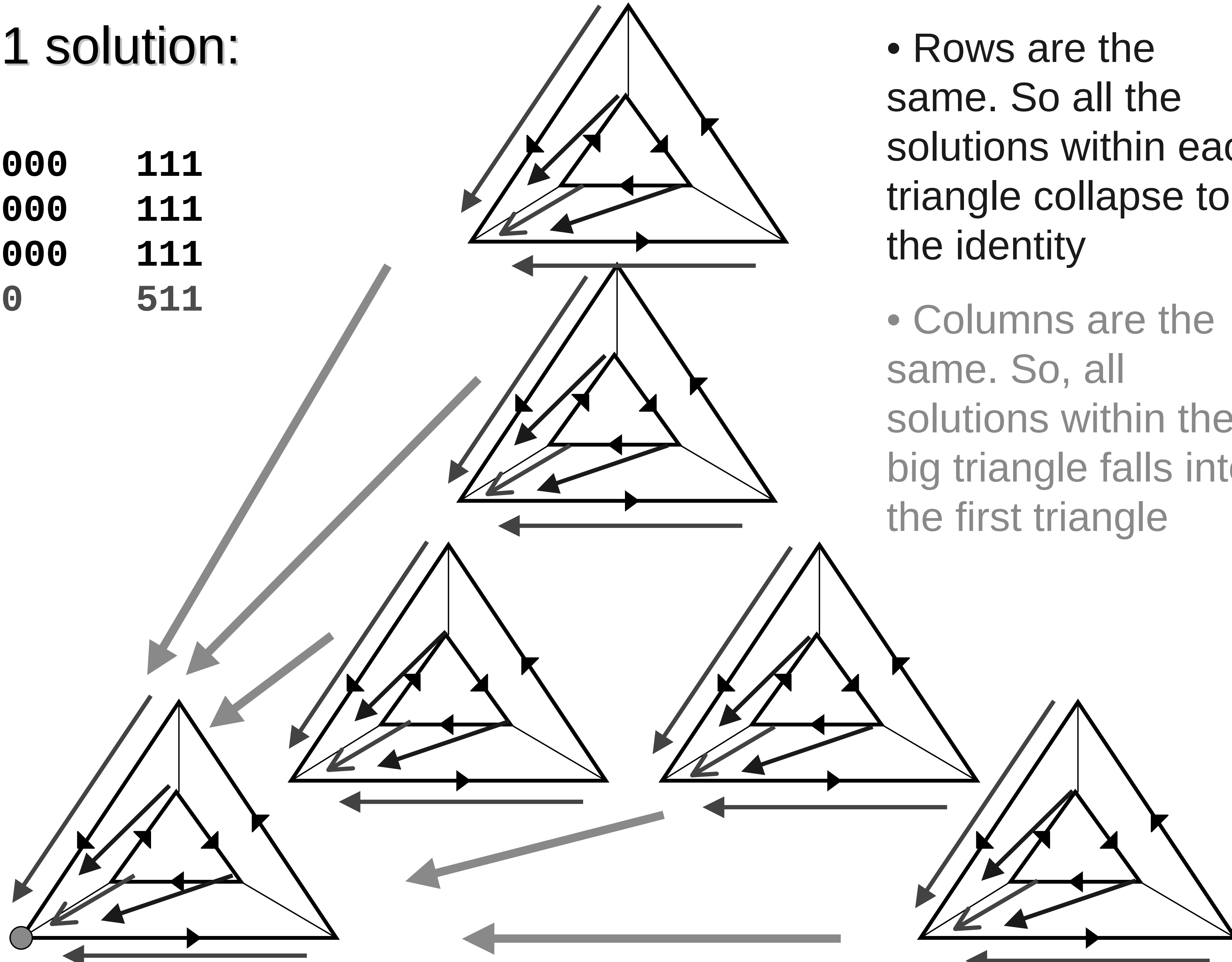
# Equivalence classes

- To understand the structure we are considering, we need to compute the size of each equivalence class.
    - *In principle each configuration has 36 equivalent states*
    - *Not all classes have the same size due to repeated elements*

- We have identified 9 possible scenarios

# 1 solution:

```
000   111
000   111
000   111
0     511
```



- Rows are the same. So all the solutions within each triangle collapse to the identity

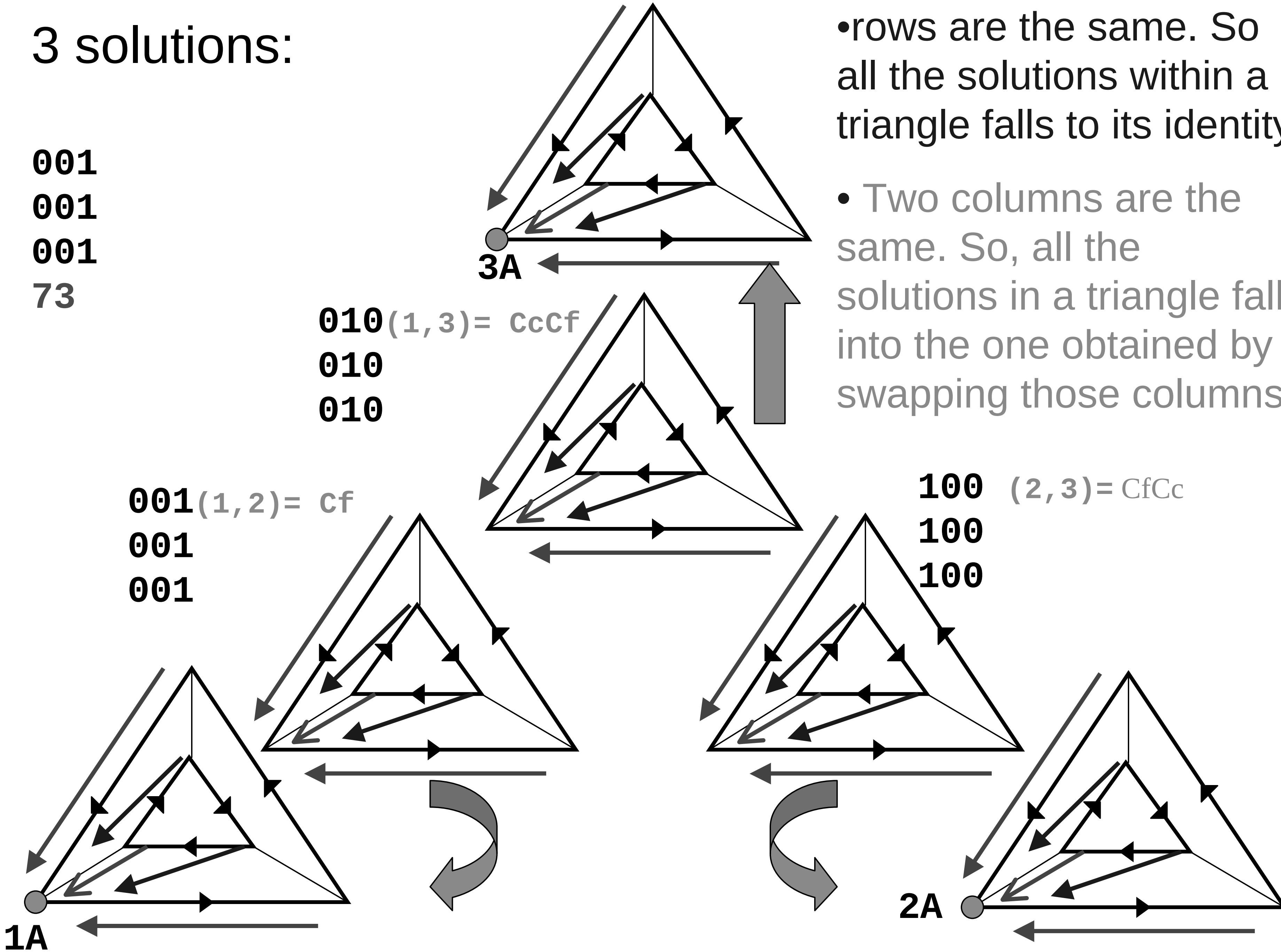
- Columns are the same. So, all solutions within the big triangle falls into the first triangle

# Size of the equivalence class

- 3 rows are equal
- 3 columns are equal
- Hence, the size is 36 / (3!3!)=1

# 3 solutions:

**001**
**001**
**001**
**73**




- rows are the same. So all the solutions within a triangle falls to its identity

- Two columns are the same. So, all the solutions in a triangle fall into the one obtained by swapping those columns.

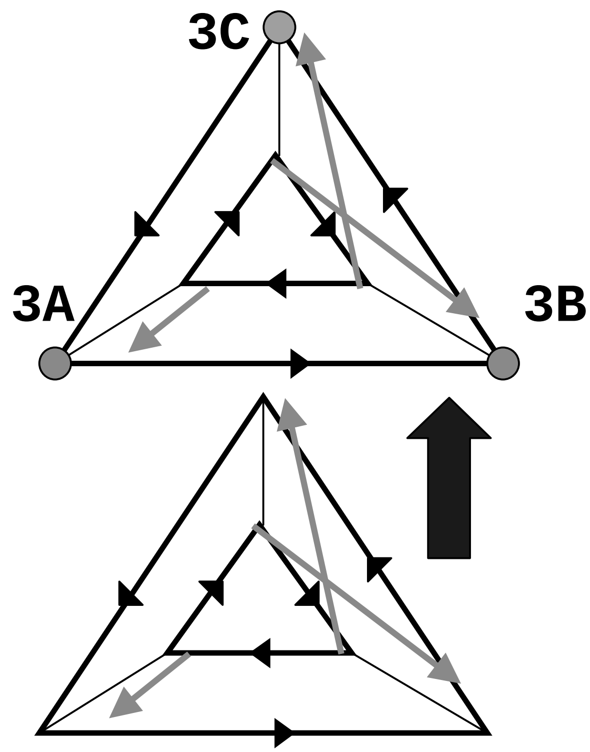
# Size of the equivalence class

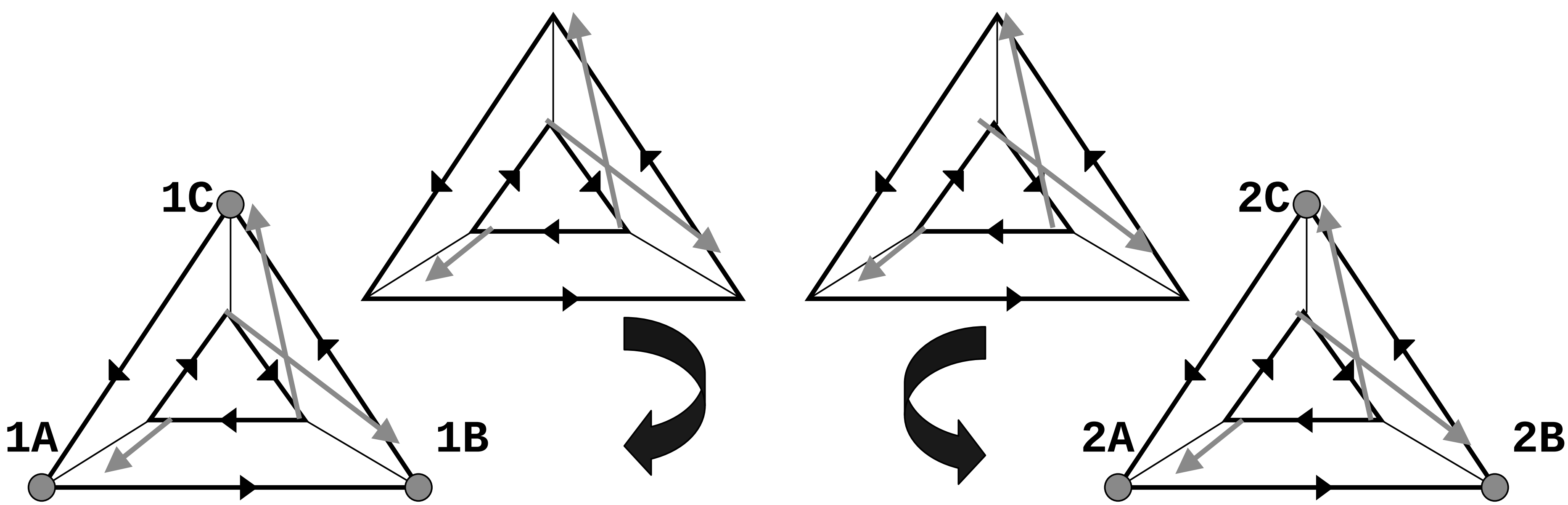
- 3 rows are equal
- 2 columns are equal
- Hence, the size is 36 / (3!2!)=3

- Same situation if
- 2 rows are equal
- 3 columns are equal

9 solutions:

**000    001**
**000    001**
**001    110**
**1       78**
**001**
**001**
**111**
**79**




•two rows are the same.

**•So, a solution in a triangle falls into the one obtained by swapping those rows.**

•moreover, two columns are the same. So, all the solutions in a triangle falls into the one obtained by swapping those columns.

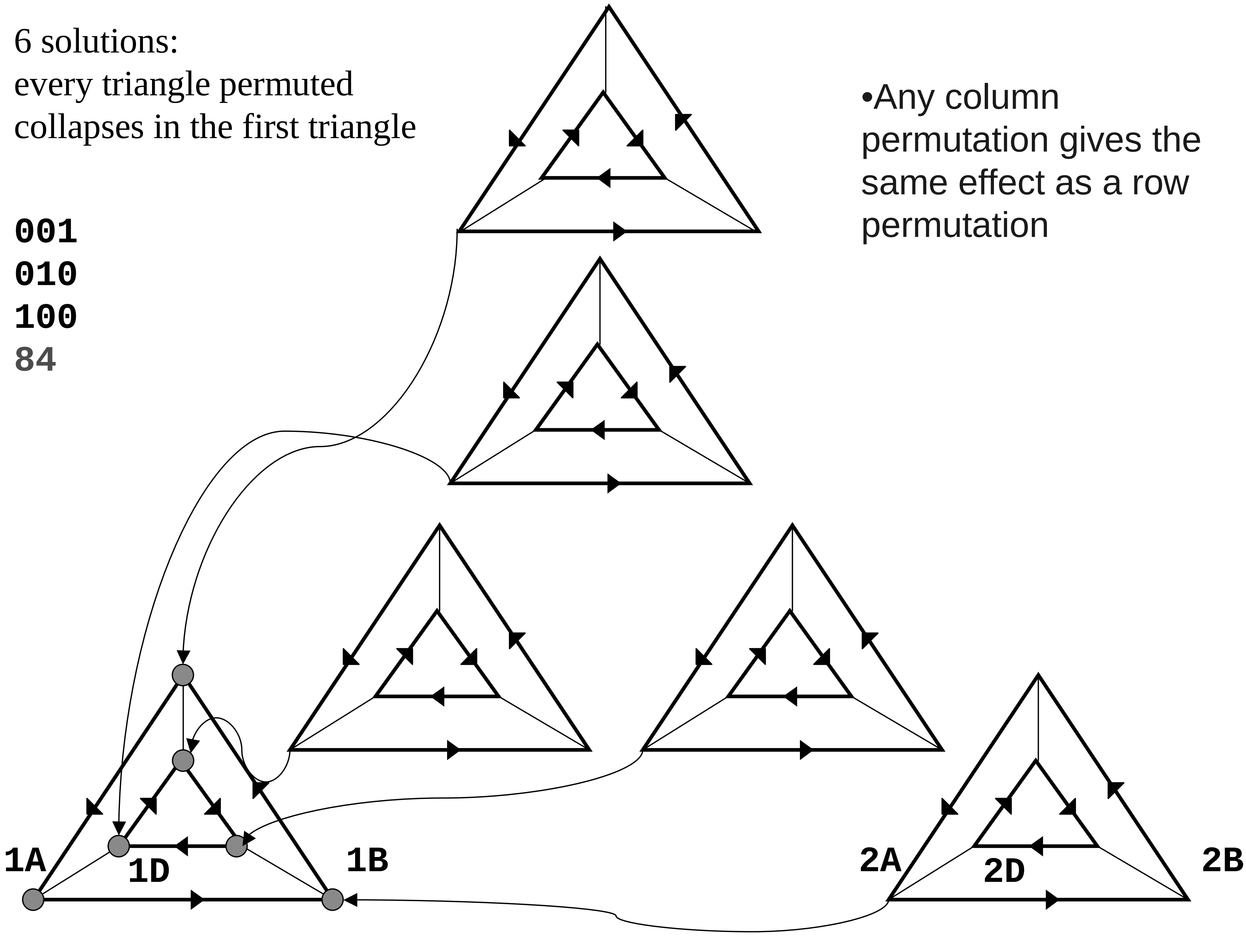
# Size of the equivalence class

- 2 rows are equal
- 2 columns are equal
- Hence, the size is 36 / (2!2!)=9

6 solutions:
every triangle permuted
collapses in the first triangle

**001**
**010**
**100**
**84**

•Any column permutation gives the same effect as a row permutation

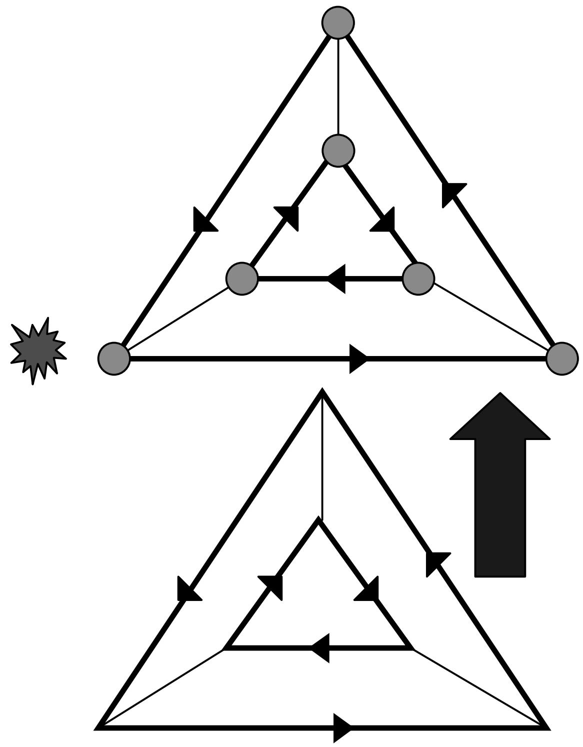


# Size of the equivalence class

- Every column permutation is equal to a row permutation.
- Hence, the size is 36 / (3!)=6

18 solutions:
every triangle permuted
collapses in one triangle

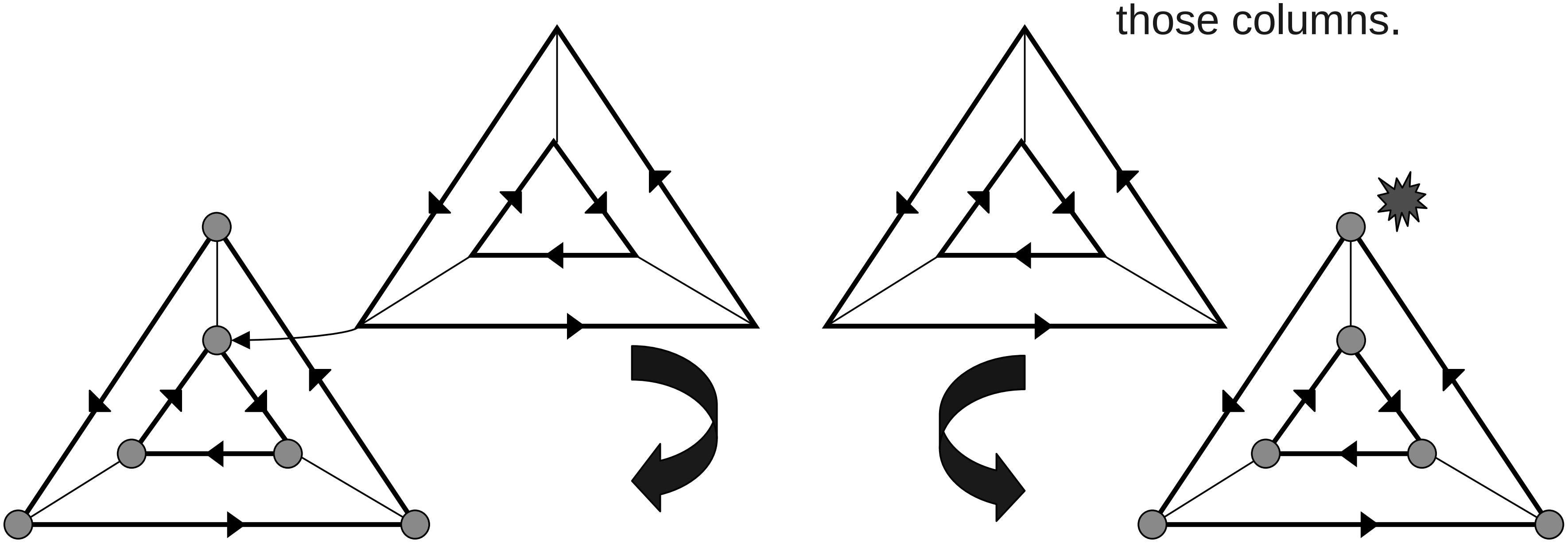
**001 000**
**011 011**
**101 101**
**93 29**



•Swapping two columns gives the same effect as swapping the last/first two rows

• OR

• two columns/rows are the same. So, all the solutions in a triangle falls into the one obtained by swapping those columns.



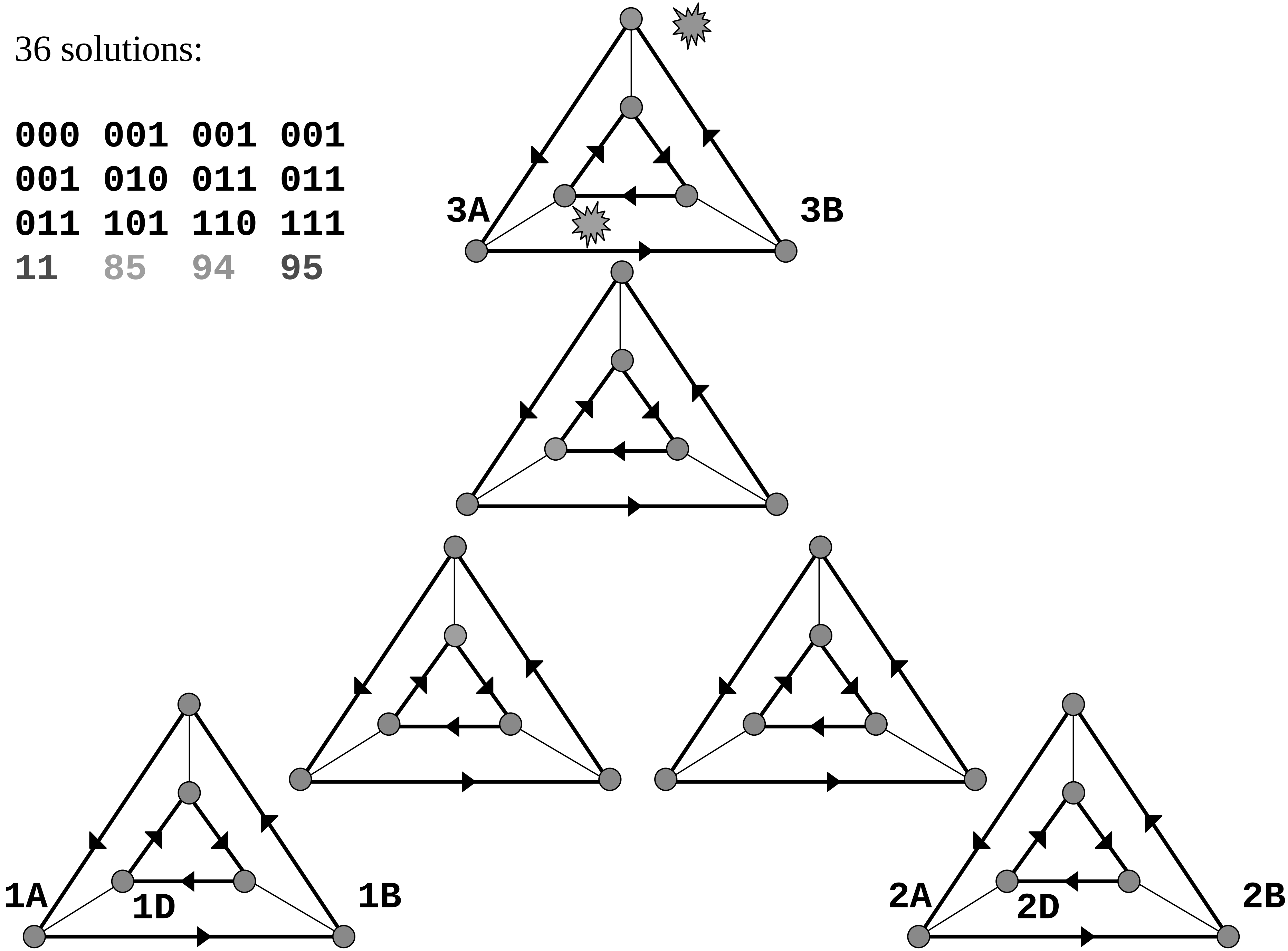
# Size of the equivalence class

- 2 rows (or columns) are equal
- Hence, the size is 36 / 2=18

- A column permutation is equal to a row permutation
- Hence, the size is 36 /2=18

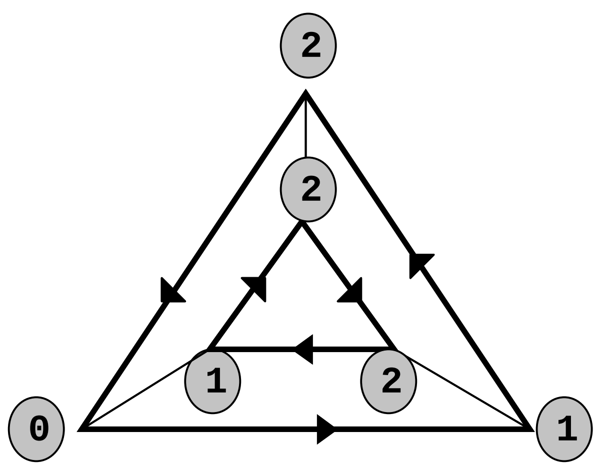
36 solutions:

**000 001 001 001**
**001 010 011 011**
**011 101 110 111**
**11 85 94 95**



# Which structure ?

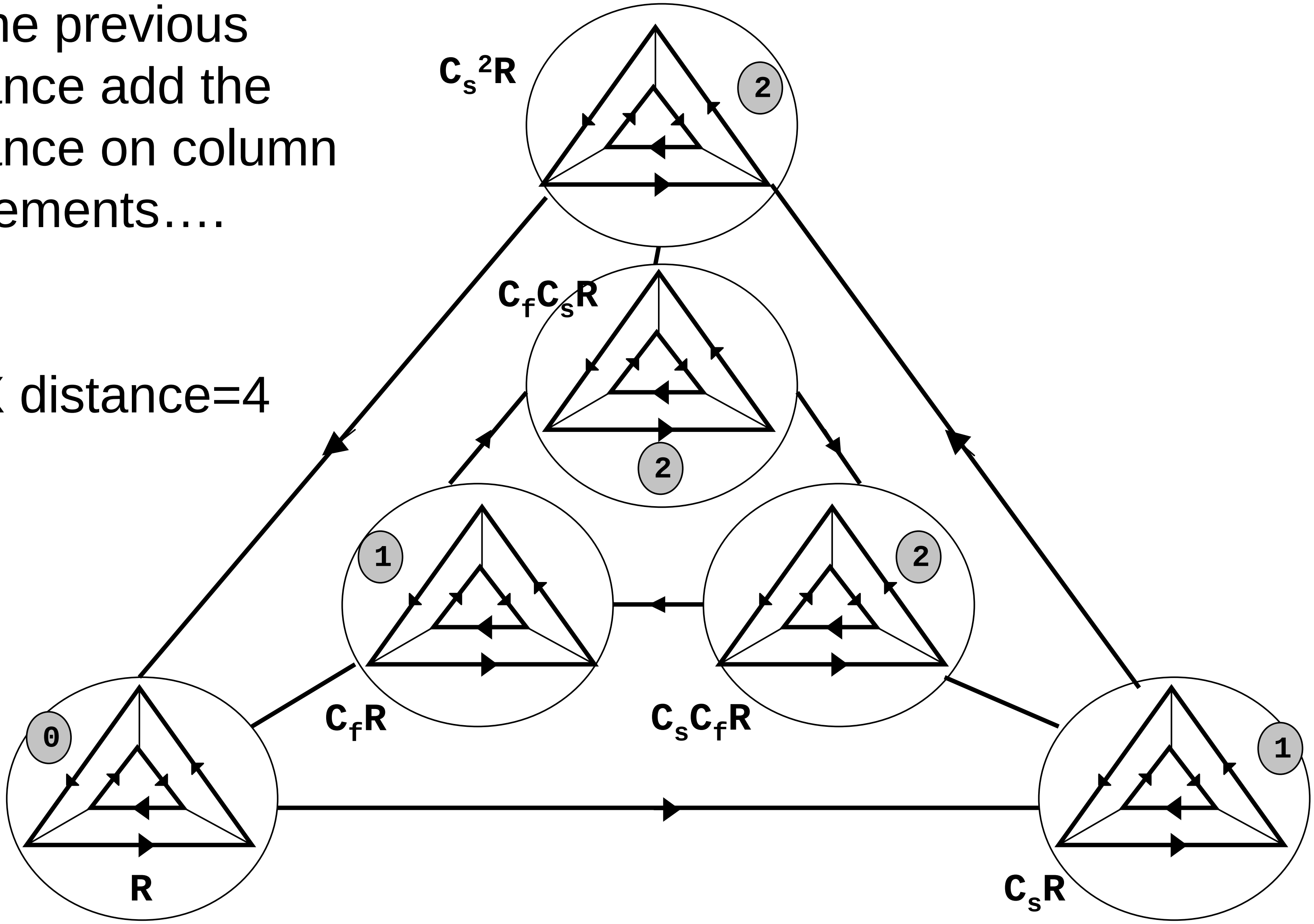
- Is there any feature or structure which can help in characterising the symmetry breaking constraint strength ?
  - *The distance ? Number of generator applications starting from I to reach a node*



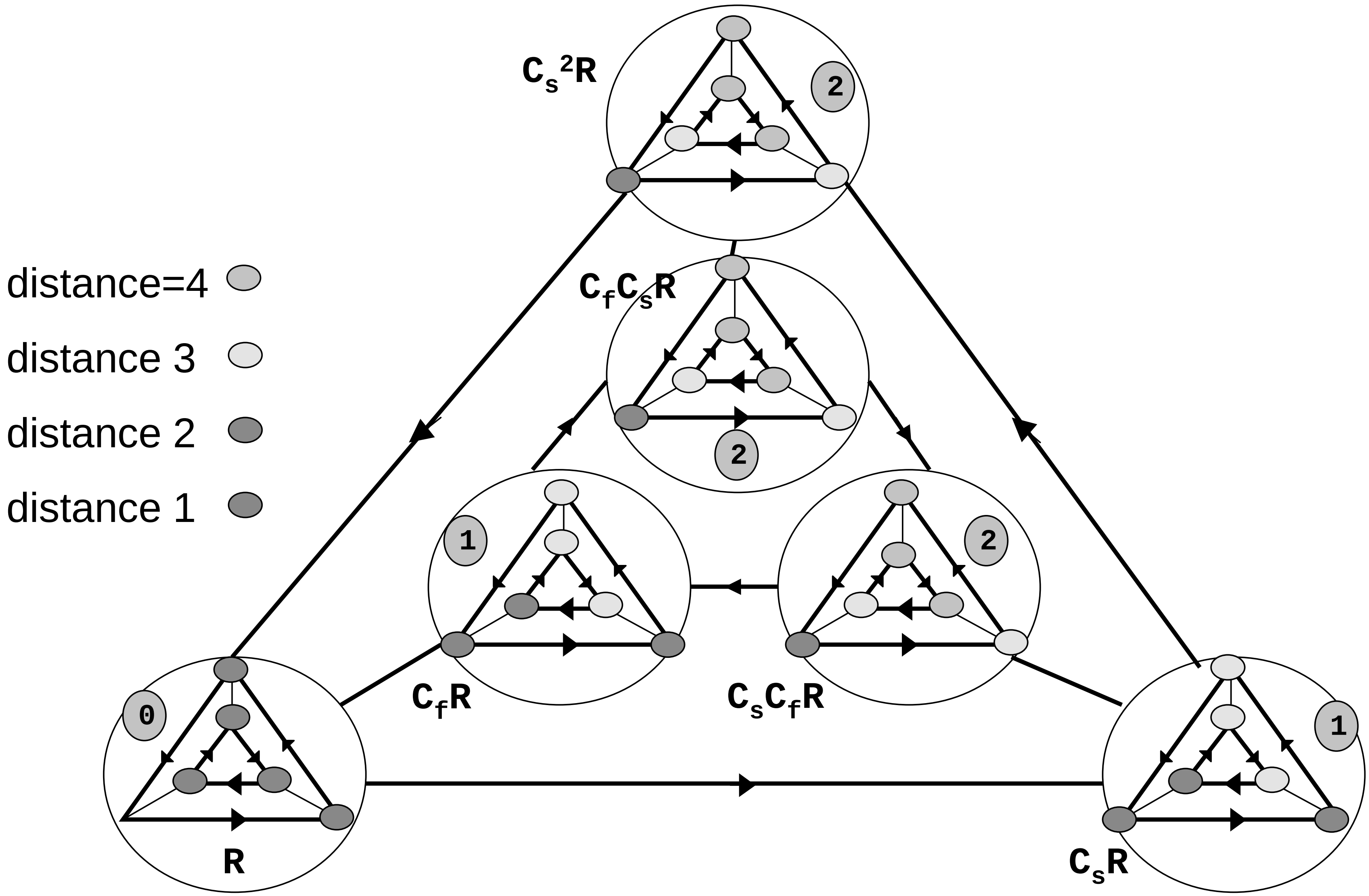
# Distance from I

To the previous distance add the distance on column movements….

MAX distance=4



# Distance from I



# Row and column lex

- All solution at distance 1 and 2 are ALWAYS removed
- Some solutions at distance 3 and 4 are not removed
- We are trying to:
  - understand why
  - define which constraints remove which solutions

# Conclusion

- Apply the method to other symmetry breaking constraints
- Define graph characteristics other then distance
- Consider graphs built from other generators