

# Breaking Row and Column Symmetries in Matrix Models

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**Abstract.** We identify an important class of symmetries in constraint programming, arising from matrices of decision variables where rows and columns can be swapped. Whilst lexicographically ordering the rows (columns) breaks all the row (column) symmetries, lexicographically ordering both the rows and the columns fails to break all the compositions of the row and column symmetries. Nevertheless, our experimental results show that this is effective at dealing with these compositions of symmetries. We extend these results to cope with symmetries in any number of dimensions, with partial symmetries, and with symmetric values. Finally, we identify special cases where all compositions of the row and column symmetries can be eliminated by the addition of only a linear number of symmetry-breaking constraints.

The full text of this statement-of-interest appears in the Proceedings of CP'02, the Eighth International Conference on Principles and Practice of Constraint Programming, Lecture Notes in Computer Science, volume 2470, Springer-Verlag, 2002.

**Acknowledgements** This work is partially supported by grant 221-99-369 of VR (the Swedish Research Council), by institutional grant IG2001-67 of STINT (the Swedish Foundation for International Cooperation in Research and Higher Education), and grant GR/N16129 of EPSRC (the UK Engineering and Physical Sciences Research Council). The last author was supported by an EPSRC advanced research fellowship. We thank our anonymous referees, Warwick Harvey and the members of the APES research group ([www.dcs.st-and.ac.uk/~apes/](http://www.dcs.st-and.ac.uk/~apes/)), especially Barbara Smith, for their helpful discussions.