Dependency-Curated Large Neighbourhood Search

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Large Neighbourhood Search (LNS)

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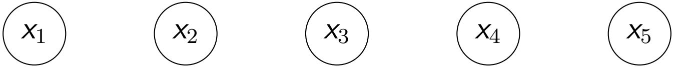

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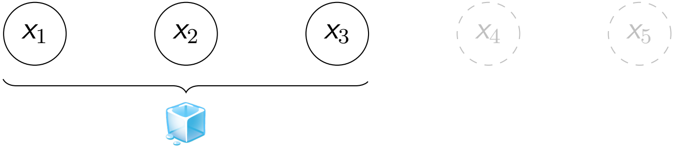
In each LNS iteration, a *freeze set* is selected, where the value of each variable in the freeze set is kept from the incumbent solution.



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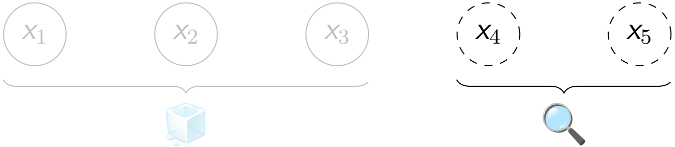
The value of each remaining variable is found and assigned via solving.



Selection Heuristics

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- The selection heuristic selects a freeze set in each LNS iteration.
- A selection heuristic can be generic or specific to a problem.
- Much research has been performed on finding LNS selection heuristics that select freeze sets that lead to high-quality solutions.

Relaxed Car Sequencing

Running example: relaxed car sequencing (RCS) description

- For each feature and any three subsequent cars, at most two of them can have that feature.
- Produce two cars of each car type.

Car Type	Red Paint	Windshield
🛻	✓	✓
🏎️	✓	✗
🚙	✗	✓

Relaxed Car Sequencing

Running example: relaxed car sequencing (RCS) description

- For each feature and any three subsequent cars, at most two of them can have that feature.
- ~~Produce two cars of each car type.~~
- Produce *at most* two cars of each non-dummy car type.
- Minimise the number of produced dummy cars.

Car Type	Red Paint	Windshield
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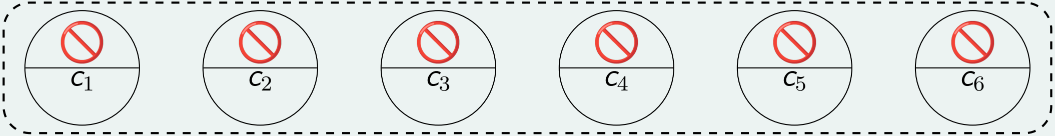
Relaxed Car Sequencing

Running example: RCS model & visualisation



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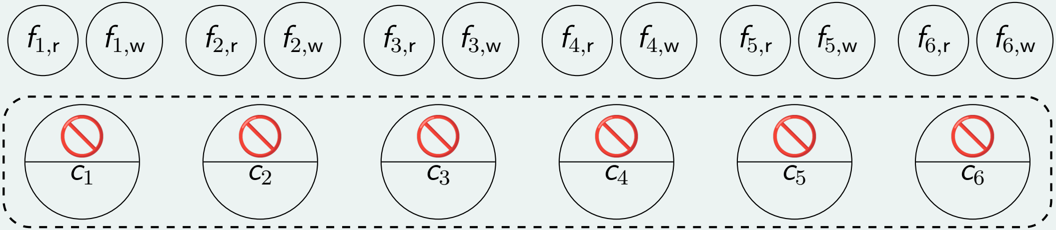

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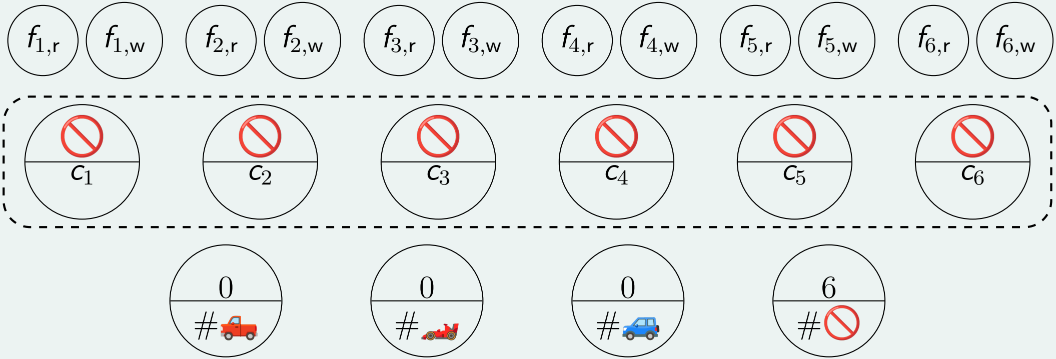

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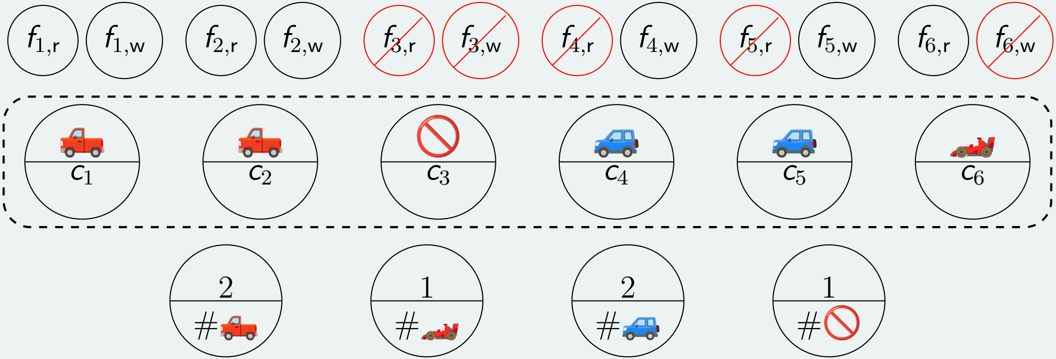

$f_{1,r}$ $f_{1,w}$ $f_{2,r}$ $f_{2,w}$ $\cancel{f_{3,r}}$ $\cancel{f_{3,w}}$ $\cancel{f_{4,r}}$ $f_{4,w}$ $\cancel{f_{5,r}}$ $f_{5,w}$ $f_{6,r}$ $\cancel{f_{6,w}}$

🛻 c_1 🛻 c_2 🚫 c_3 🚙 c_4 🚙 c_5 🏎️ c_6

2 #🛻 1 #🏎️ 2 #🚙 1 #🚫

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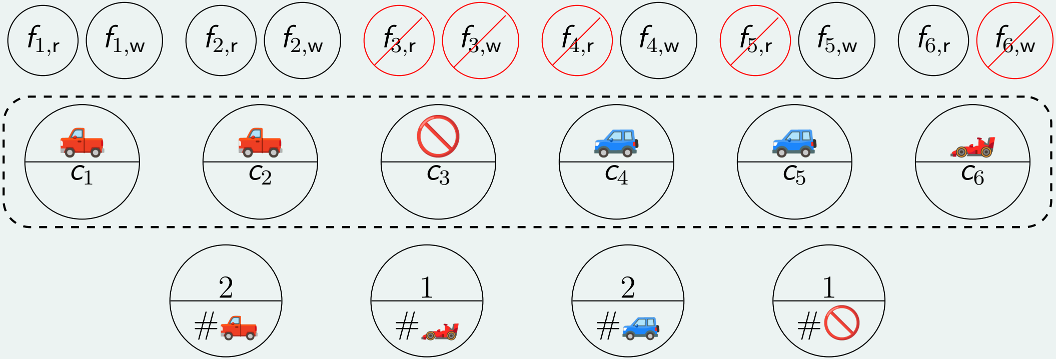

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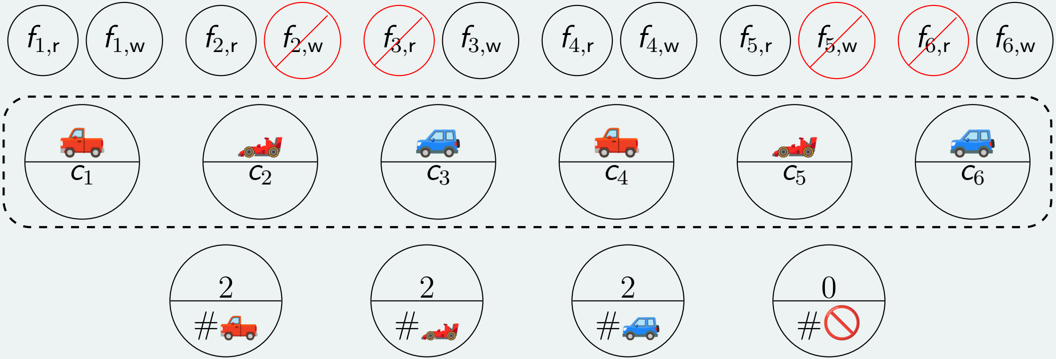
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c_1
c_2
c_3
c_4
c_5
c_6
2 #
2 #
2 #
0 #

Any assignment of the variables $\{c_1, \dots, c_6\}$ fixes the values of the remaining variables.

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Example: dependency constraint

The number of variables in $\{c_1, \ldots, c_6\}$ that take value 🛻.

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For a generic selection heuristic, *any* variable can typically be included in the freeze set.

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- The set of all variables is a set of search variables.

Approach

1. Before search starts, find a set $\mathcal{S}$ of search variables.

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2. During each LNS iteration, enforce the freeze set to be a subset of $\mathcal{S}$.

The Dependency Graph

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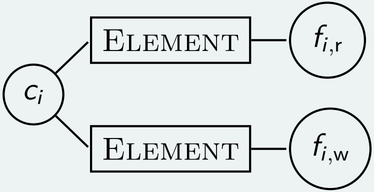
- For each dependency constraint with input variables $\mathcal{X}$ and output variables $\mathcal{Y}$, there is a vertex d in the graph and
  - an arc from each $x \in \mathcal{X}$ to d and
  - an arc from d to each variable $y \in \mathcal{Y}$.

Example: the Dependency Graph

Running example: RCS dependency subgraph



Note that not all variables, arcs, and edges are depicted in the subgraph.

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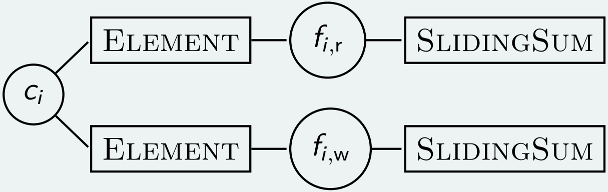

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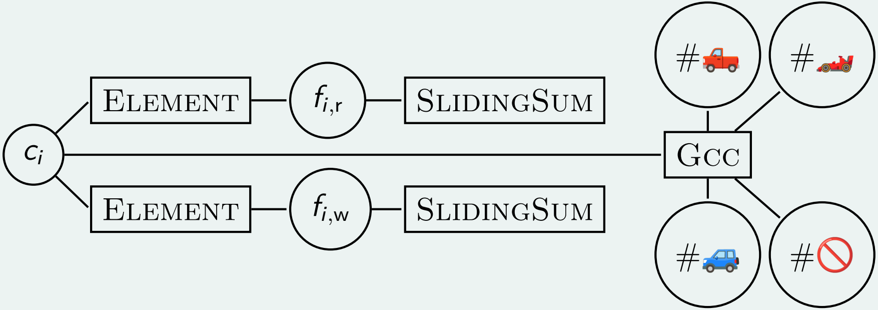

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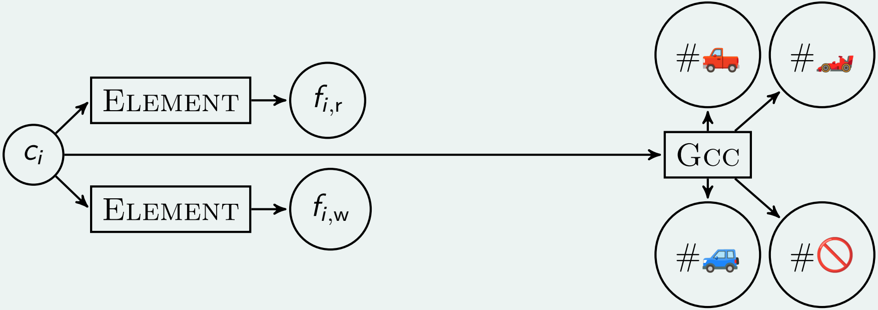

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The Dependency Curation Scheme (DCS)

We have developed a *dependency curation scheme* (DCS), that finds a low-cardinality set of search variables.

- If the dependency graph is acyclic, then the set $\mathcal{S}$ of search variables is the set of source variables.

The Dependency Curation Scheme (DCS)

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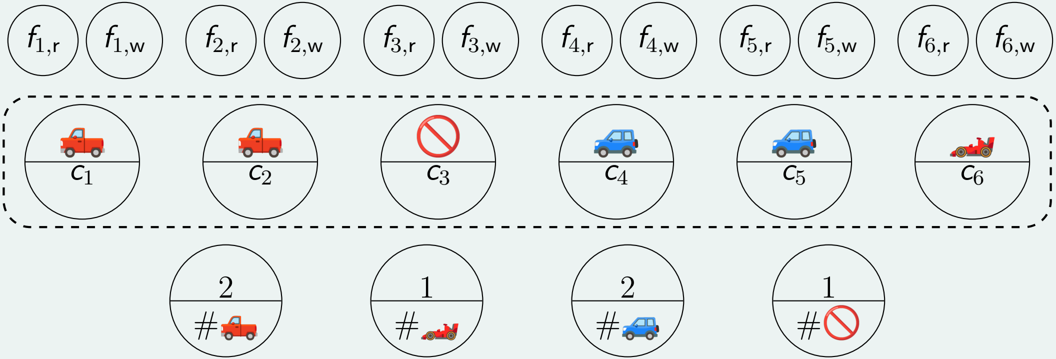
- If the dependency graph is acyclic, then the set $\mathcal{S}$ of search variables is the set of source variables.
- Otherwise, we find a set of search variables using an algorithm that:

  1. finds the strongly connected components (SCCs) and orders them, before it
  2. constructs $\mathcal{S}$ while it explores the ordered SCCs in a manner similar to depth-first search.

The Dependency Curation Scheme (DCS): Example

Running example: RCS set of search variables



$f_{1,\mathrm{r}}$ $f_{1,\mathrm{w}}$ $f_{2,\mathrm{r}}$ $f_{2,\mathrm{w}}$ $f_{3,\mathrm{r}}$ $f_{3,\mathrm{w}}$ $f_{4,\mathrm{r}}$ $f_{4,\mathrm{w}}$ $f_{5,\mathrm{r}}$ $f_{5,\mathrm{w}}$ $f_{6,\mathrm{r}}$ $f_{6,\mathrm{w}}$

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2 #🛻 1 #🏎️ 2 #🚙 1 #🚫

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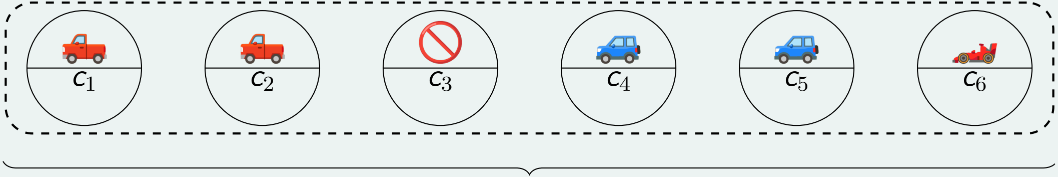

🧊 freeze set must be a subset of this set

Helicopter View

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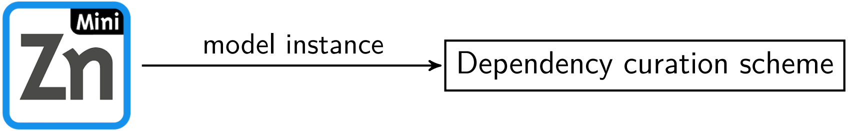
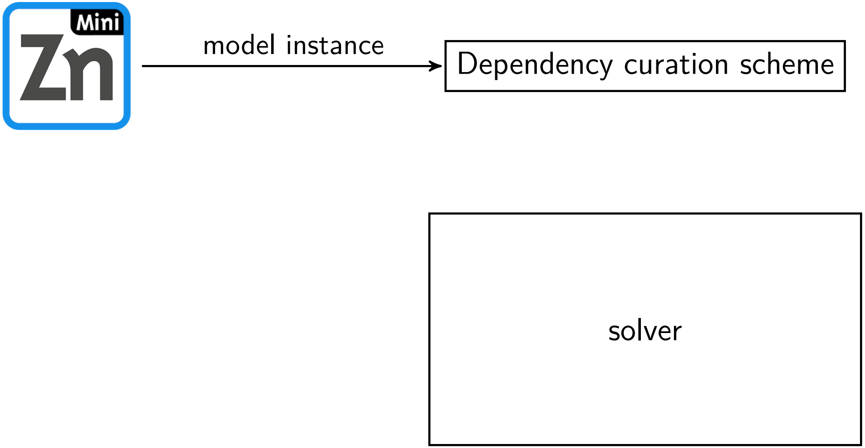


model instance

Dependency curation scheme

Helicopter View



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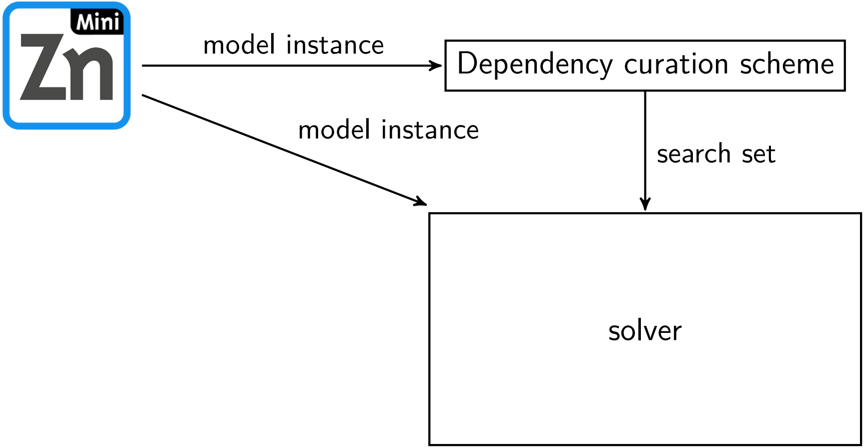
Dependency curation scheme

solver

Helicopter View



model instance

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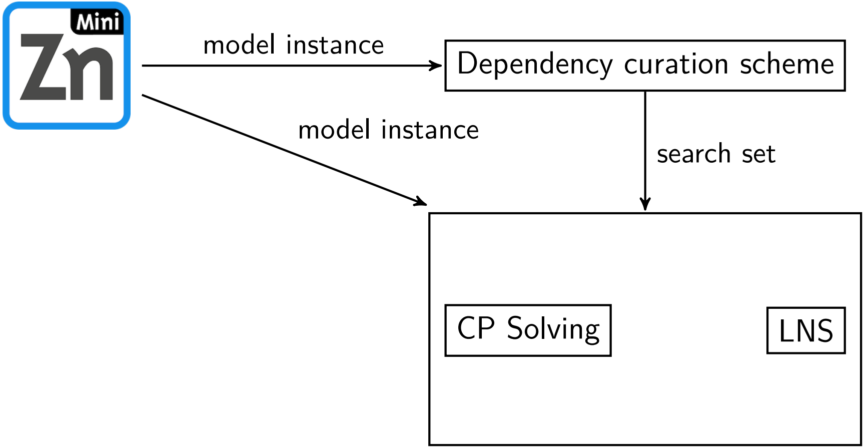

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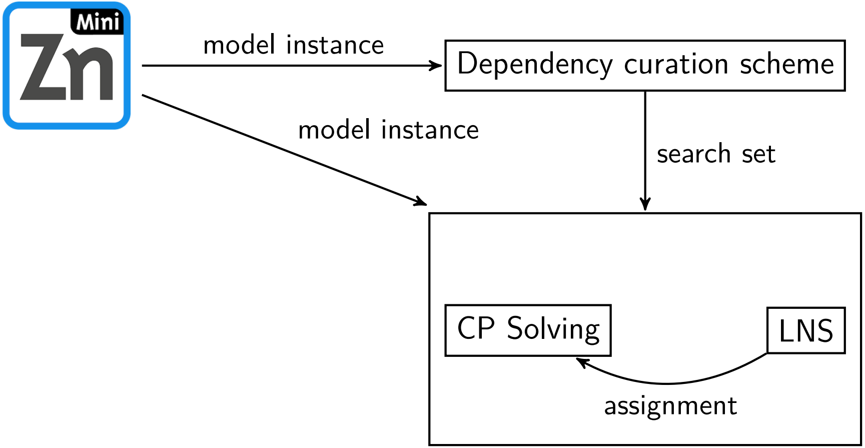

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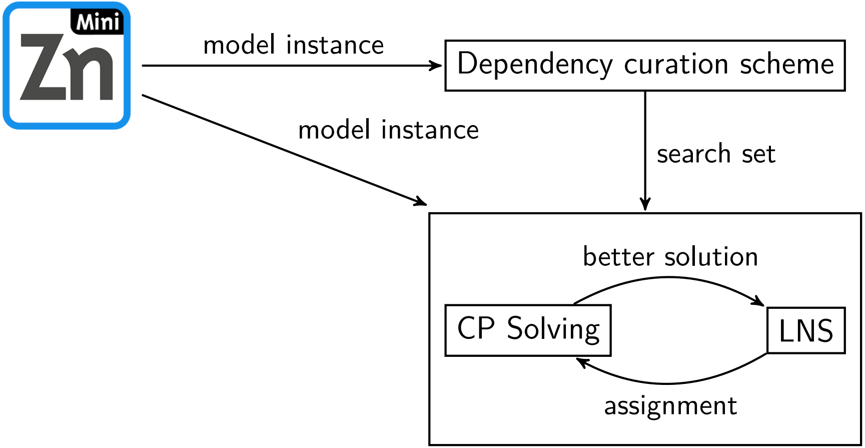

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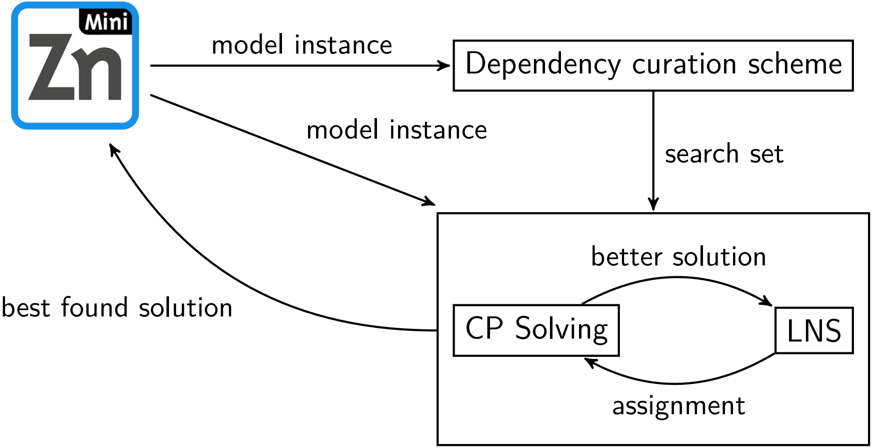

LNS selection heuristics

We selected five generic LNS selection heuristics:

- Randomised LNS;
- Propagation guided LNS (PG-LNS);
- Reverse propagation guided LNS (RPG-LNS);
- Cost impact guided LNS (CIG-LNS); and
- Variable-relationship guided LNS (VRG-LNS).

Randomised LNS

We have a threshold $\phi \in [0.05, 0.95]$ that is updated during search.

Inside every LNS iteration and for each variable x in the set of search variables, we select a uniform random number $p_x \in [0, 1]$.

If $p_x < \phi$, then x is included in the freeze set, else it is excluded.

Remaining Selection Heuristics

Each of the other generic selection heuristics stores data about the search variables.

The data is collected during LNS iterations and is exploited to help guide search.

Method

We extended a Gecode-based portfolio solver, implementing or reimplementing its generic selection heuristics to follow their descriptions from the literature more closely.

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We created or updated existing MiniZinc models for the:

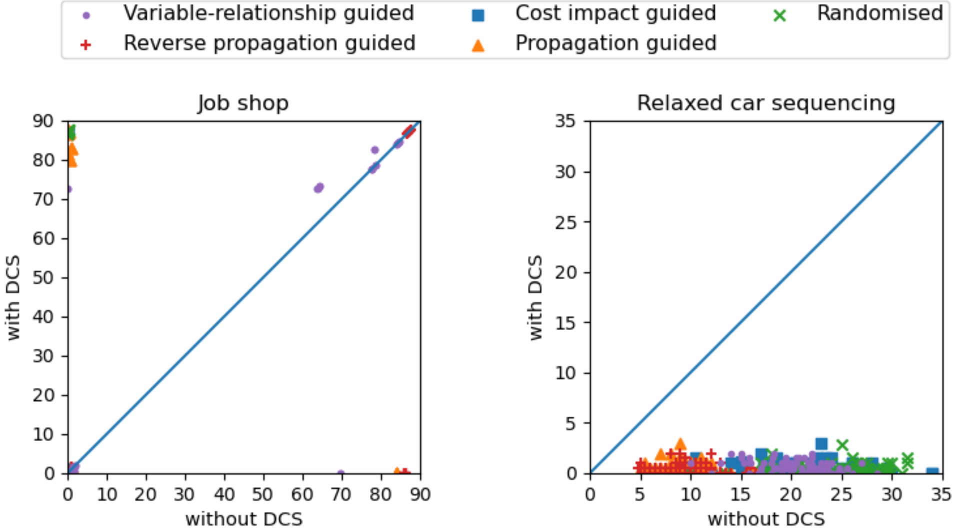
- job shop problem (JSP),
- steel mill slab design (SMSD),
- relaxed car sequencing problem (RCS), and
- travelling salesperson with time windows (TSPTW).

Experiment Setup

For each selection heuristic, both with DCS and without DCS, and each problem instance, we ran 10 independent runs, each under a timeout of 3 minutes from the same initial incumbent solution.

We used a scoring function where scores range over the interval $[0, 100]$, where a low score is better than a high one.

Results – Graphs



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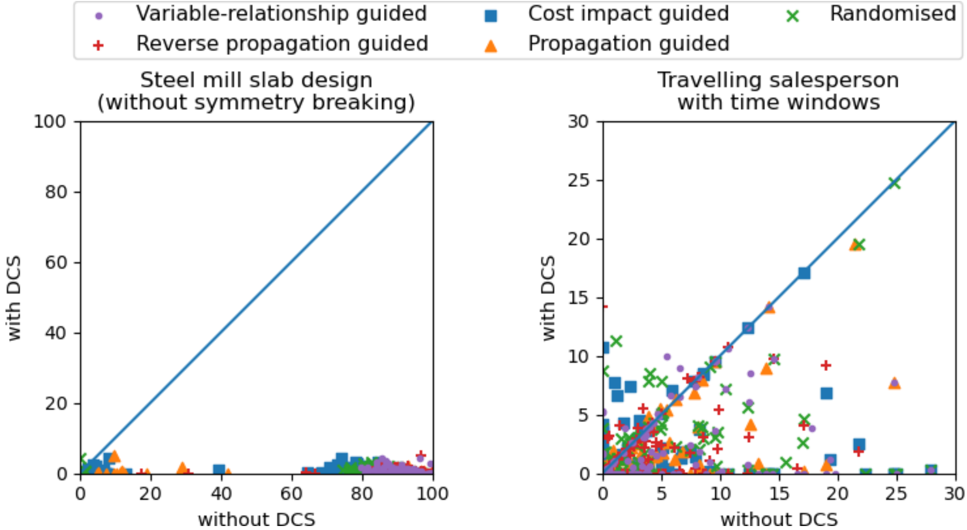

Results – Table

	CIG-LNS		PG-LNS		Randomised LNS		RPG-LNS		VRG-LNS	
DCS	no	yes	no	yes	no	yes	no	yes	no	yes
JSP	**0.13**	7.45	7.30	28.53	0.18	18.37	32.68	18.35	32.20	33.19
RCS	22.11	0.47	9.94	0.48	24.72	**0.34**	9.93	0.43	20.35	0.63
SMSD	1.62	0.87	4.06	**0.67**	1.14	0.85	5.64	0.73	6.45	0.81
TSPTW	4.65	1.82	3.67	**1.36**	4.87	2.18	3.71	1.80	5.41	2.20

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We have presented our dependency curation scheme (DCS), which can be used with any LNS selection heuristic.

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We have compared the performance of a generic randomised selection heuristic and state-of-the-art generic selection heuristics, both with and without DCS.

Our experiments show that the performance of using DCS with the generic randomised selection heuristic is (amazingly) competitive.

Thank You

Thank you for your attention.

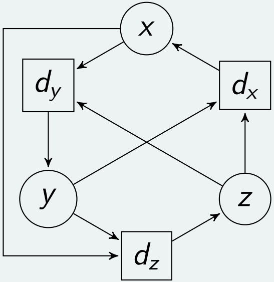
Multi-way Dependency Constraints

Some constraints have multiple input sets and multiple output sets. Each such constraint is represented as multiple vertices in the dependency graph.

Example: multi-way dependency constraint

The constraints $x + y = z$ can be rewritten:

- $z - y = x$ and
- $z - x = y$.



Conditional Dependency Constraints

Some constraints are dependency constraints under specific circumstances.

Example: the table constraint

For each car i, We can replace the element constraints with a table constraint:

$$\texttt{table([}c_i\texttt{, }f_{i,\mathrm{r}}\texttt{, }f_{i,\mathrm{w}}\texttt{],}\quad \begin{array}{l}\texttt{[[🛻, 1, 1],}\\ \texttt{ [🏎️, 1, 0],}\\ \texttt{ [🚙, 0, 1],}\\ \texttt{ [🚫, 0, 0]]}\end{array}\quad\texttt{)}$$

For the parameter matrix, the values in the first column are distinct. Therefore, this table constraint is a dependency constraint via which c_i functionally defines $f_{i,\mathrm{r}}$ and $f_{i,\mathrm{w}}$.