Seeking Practical CDCL Insights from Theoretical SAT Benchmarks

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The SAT Problem

- Literal a: Boolean variable x or its negation $\overline{x}$ (or $\neg x$)
- Clause $C = a_1 \vee \cdots \vee a_k$: disjunction of literals
  (Consider as sets, so no repetitions and order irrelevant)
- CNF formula $F = C_1 \wedge \cdots \wedge C_m$: conjunction of clauses

Has F satisfying assignment?

The Power of so called CDCL SAT Solvers

2017 SAT Competition [BHJ17]

- largest solved benchmark (g2-T96.1.1.cnf)
  - 8 905 808 variables
  - 32 322 587 clauses
  - verifiable UNSAT in 4126.12s
- smallest unsolved (mp1-bsat222-777.cnf)
  - 222 variables
  - 777 clauses
  - timelimit 5000s

Explanation?

Understanding Performance

Problem instance determines:

- solver performance
- which algorithms / heuristics are important / good

Solvers essentially do resolution
$\Rightarrow$ well understood through proof complexity

- scalable UNSAT problems
- extremal w.r.t. certain property
  $\Rightarrow$ lower bound on runtime
- expect different behaviour

Our Project

Goal:

- understand which / when settings are important

Our approach for reaching this goal:

- crafted benchmarks[1], using knowledge from proof complexity
- benchmarks are
  - scalable
  - easy
  - extremal (or close to)
- instrument solver to switch between algorithms / heuristics

[1] generated using CNFGen [LENV17]

Related Work

- instrumentation [LM02, KSM11]

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Related Work

- instrumentation [LM02, KSM11]
- decision heuristics [BF15]
- restart schemes [Hua07]

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- resolution space on theory formula [JMNŽ12]

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The CDCL Algorithm [DP60, DLL62, MS99, MMZ+01, ...]

Used Implementations: MiniSat [ES04], Glucose [AS09]

```
procedure SOLVE(F)
    while v ← next variable decision do
        assign v to chosen phase
        do unit (fact) propagation
        if conflict then
            add clause learned from conflict
            if decision to be undone then undo bad decisions
            else return UNSAT

    return SAT
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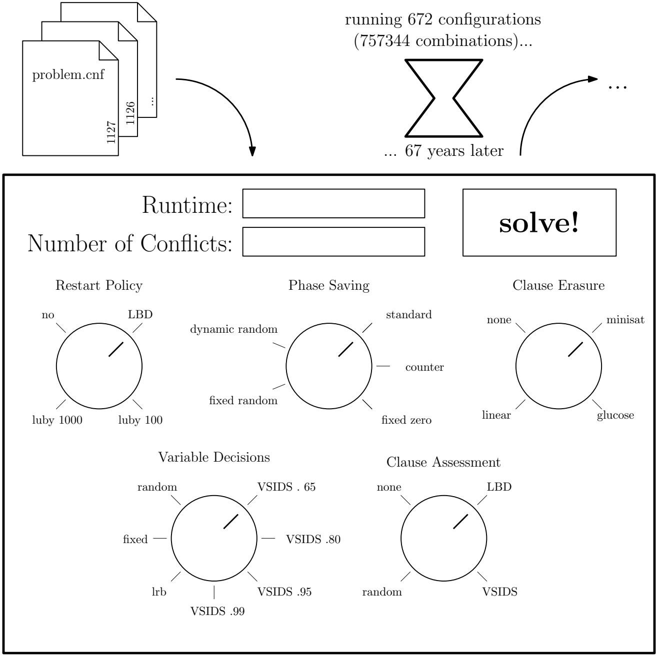
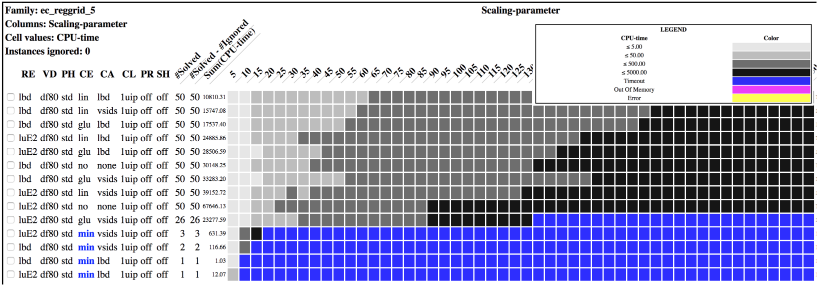

problem.cnf
1127
1126
...
running 672 configurations
(757344 combinations)...
... 67 years later
...
Runtime:
Number of Conflicts:
solve!
Restart Policy
no
LBD
luby 1000
luby 100
Phase Saving
dynamic random
standard
counter
fixed random
fixed zero
Clause Erasure
none
minisat
linear
glucose
Variable Decisions
random
VSIDS . 65
fixed
VSIDS .80
lrb
VSIDS .99
VSIDS .95
Clause Assessment
none
LBD
random
VSIDS

Heatmaps



- row: setting
- column: scaled instances
- colour: runtime

Analysing PAR-Score

PAR-X-score: runtime if solved, otherwise $X \cdot$ timelimit
($X = 2$ used)

Analyse:

- fix some “knobs”
- compute expected score
  (average of settings containing fixed “knobs”)
- compare to global average, but:
  - always some difference
  - choose random subset of settings
    $\Rightarrow$ yields standard deviation
    (used to “value” expected score)

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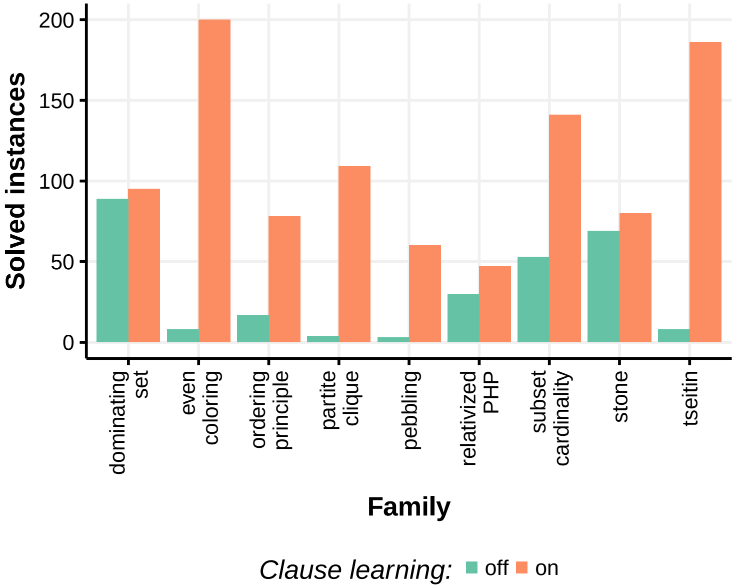
Clause Learning, Going Beyond Treelike Resolution



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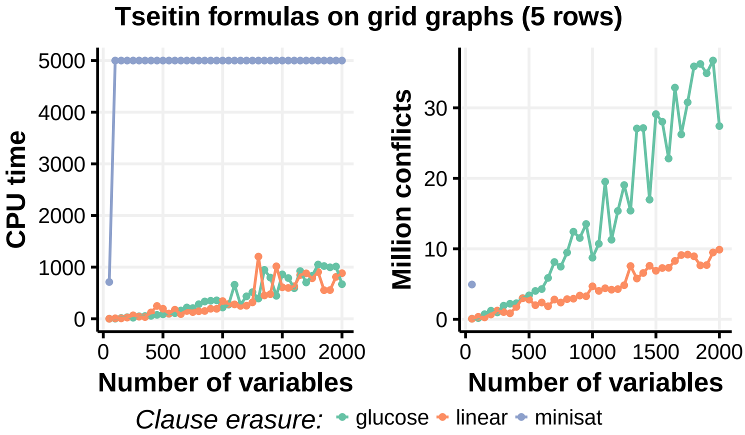
DB Size on Theoretical Time-Space Trade-Off Formulas



database size: minisat $<$ glucose $<$ linear

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procedure SOLVE(F)
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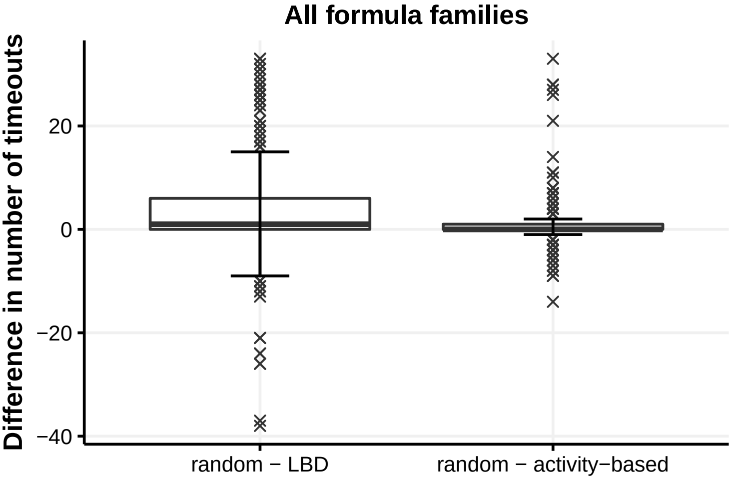
Clause Assessment



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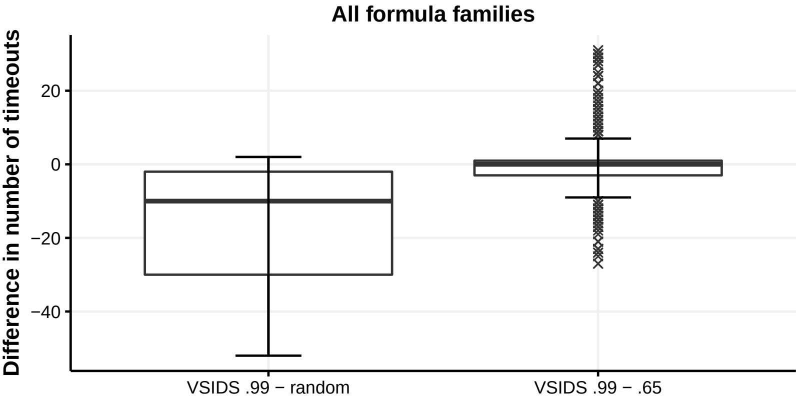
Variable Decision



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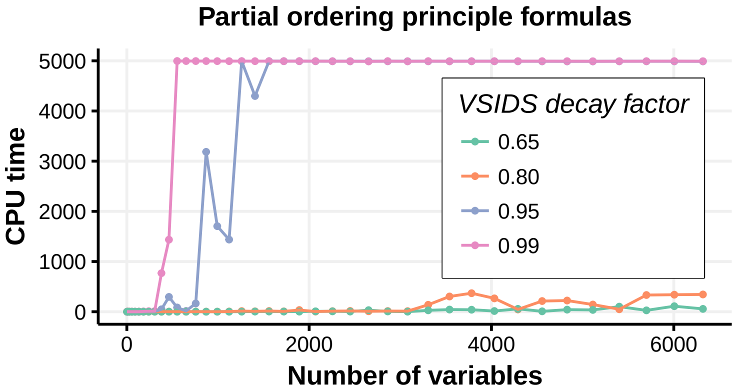

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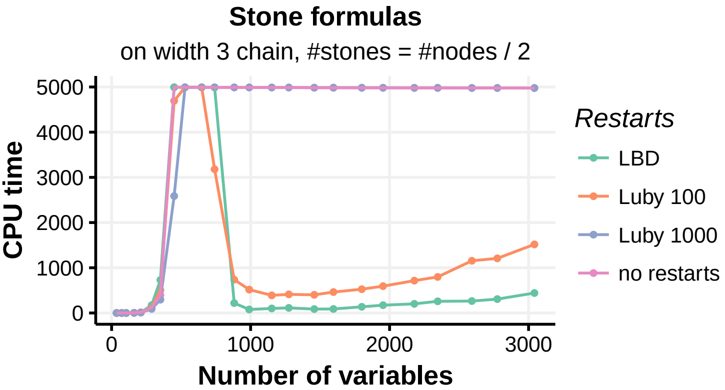
Restarts for Unrestricted Resolution



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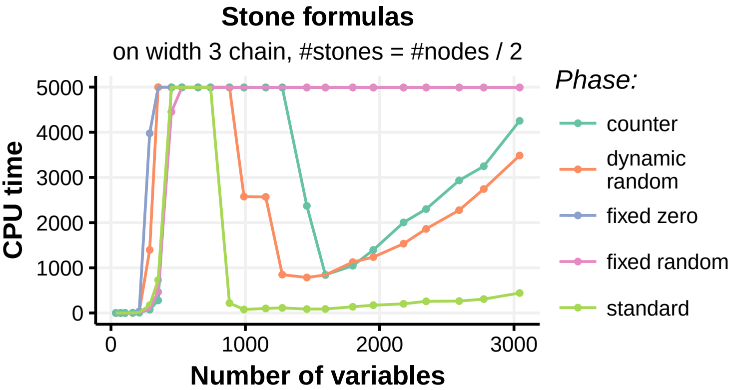
Phase Saving



Conclusions

- clause learning is important
  (if you need to go beyond treelike resolution)
- choose the *right* database size
  (required space vs. overhead)
- restarts help to harness the full power of resolution
  (if necessary)
- VSIDS is good for variable decisions
  (but can go badly wrong)

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Thank you for your attention!

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