DUCT: An Upper Confidence Bound Approach to Distributed Constraint Optimisation Problems

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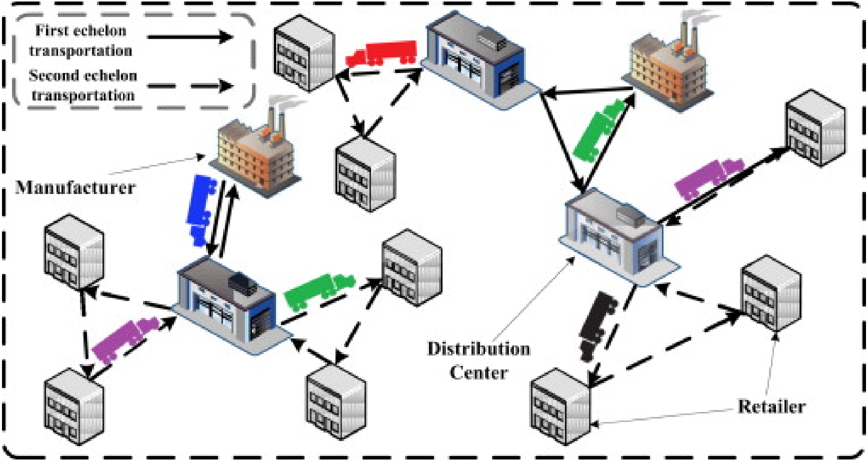
Examples

	Tuesday		
	2-3pm	3-4pm	4-5p
Michael	✓	✓	✗
Oliver	✗	✓	✓
Lily	✓	✓	✗
Sophia	✗	✓	✗

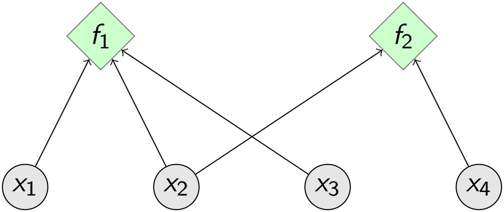
Examples


First echelon transportation
Second echelon transportation
Manufacturer
Distribution Center
Retailer

DCOP: Distributed constrained optimisation

- Variables $\mathcal{X} \triangleq \{x_1, \ldots, x_N\}$.
- Factors $\mathcal{F} \triangleq \{f_1, \ldots, f_M\}$

$$f(\mathbf{x}) \triangleq \sum_{i=1}^{M} f_i(\mathbf{x}_i) < \infty, \quad (1)$$

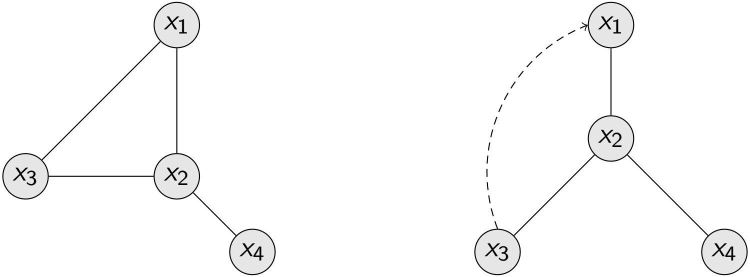


The optimisation problem

$$\min_{\mathbf{x}} f(\mathbf{x})$$

Communication

Assume each variable x_i is controlled by a single agent i.

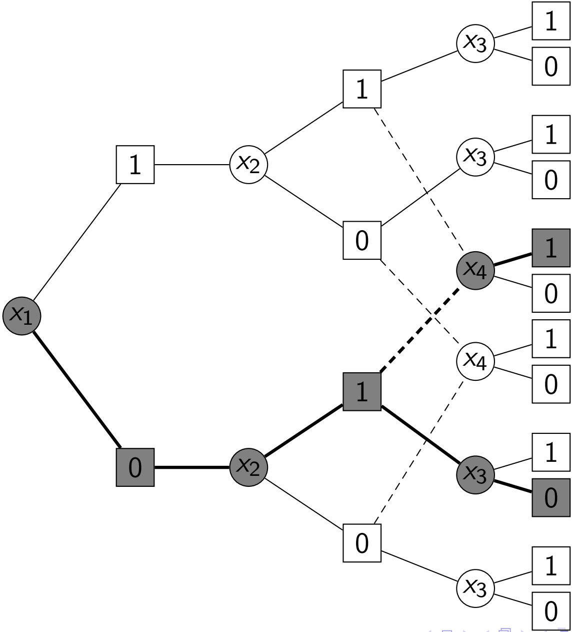


(a) Constraint graph

(b) pseudo-tree

- Constraints are represented by edges in the constraint graph.
- Tree represents information flow, including back-edges.

Search graph



Random sampling

Algorithm 1: $\texttt{SAMPLE}(a, k)$: random sampling

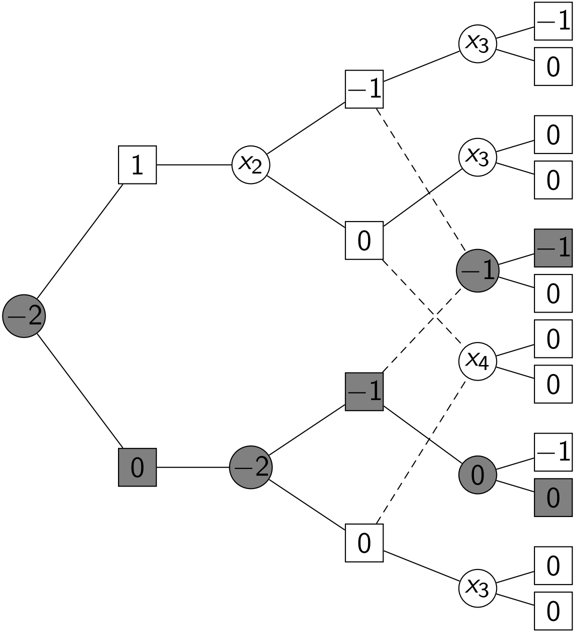
d = random value from D_k;
while $\ell^k(a, d) = \infty$ *and there are untried values* **do**
    d = random value from D_k;
return d

Cost propagation

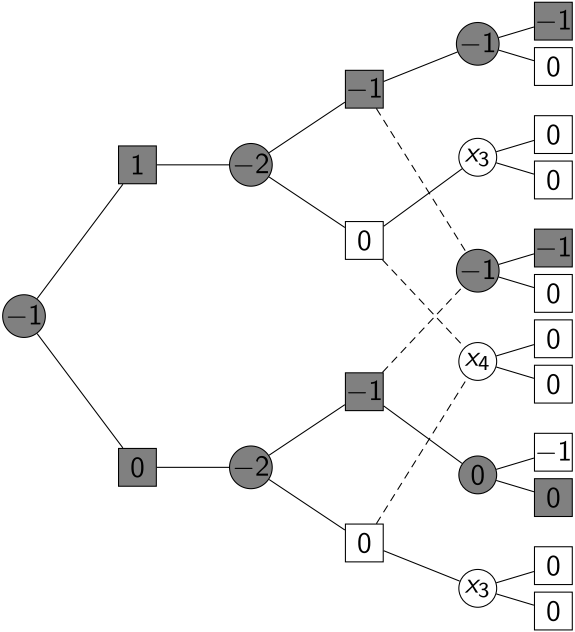
$$y_k^t = \min_{d \in D_k} \ell^k(a, d)\,, \tag{2}$$

$$y_k^t = \ell^k(a, d) + \sum_{k' \in \mathcal{C}_k} y_{k'}^t\,. \tag{3}$$

Value propagation



Value propagation



DUCT sampling

Idea: Optimistic sampling

For each node i, choose option c minimising

$$B_{i,c} \triangleq \max \left\{ \mu_{i,c} - L_{i,c},\ \ell(i, c) + \sum_k B_k \right\}, \tag{4}$$

Confidence bounds

- μ: average cost so far for (i, c).
- $L \to 0$ as we explore i, c more.
- If children disagree, ignore L.

DUCT sampling

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Analysis

Definition

Let r_t be the *instantaneous regret*

$$r_t \triangleq y_{\mathcal{D}}^t - f^*(\mathcal{D}), \tag{5}$$

and ρ_T be the *(simple) regret* after T steps:

$$\rho_T \triangleq \min\left\{ r_t \mid t = 1, \ldots, T \right\}. \tag{6}$$

i.e. the ϵ-optimality of the best solution till time T.

Assumptions

Assumption

The number of very bad solutions within any solution subset A is small.

$$\lambda\left(\{\mathbf{x} \in A \,|\, f(\mathbf{x}) > f^*(A) + \epsilon\}\right) \leq \lambda(A)\,\gamma\epsilon^{-\beta}. \tag{7}$$

Assumption

The set of optimal solutions has non-zero measure, i.e.

$$\lambda^* \triangleq \lambda\left(\mathcal{D}^*\right) > 0, \tag{8}$$

Results

Theorem

A lower bound on the expected regret is $\mathbb{E}\,\rho_T \in \tilde{\Omega}(e^{-T})$.

Theorem

For the random algorithm

$$\mathbb{E}\,\rho_T \in \tilde{O}(1/T + e^{-T}) \tag{9}$$

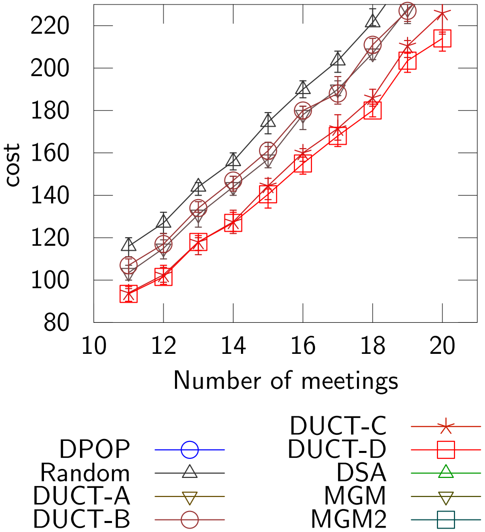
Theorem

For DUCT

$$\rho_T \leq \tilde{O}(1/\Delta T + e^{-T}), \tag{10}$$

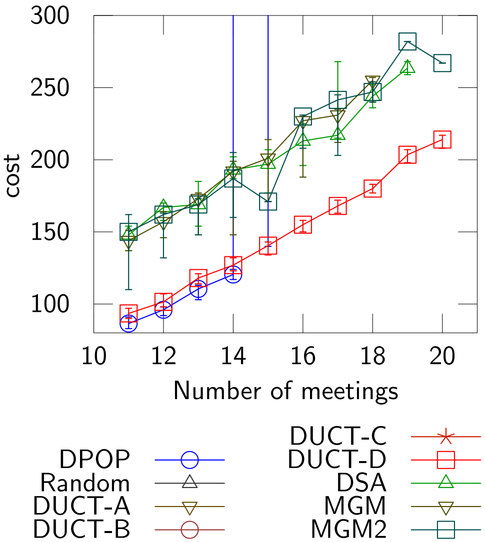
where Δ *captures how easy it is to distinguish a good branch.*

Meeting

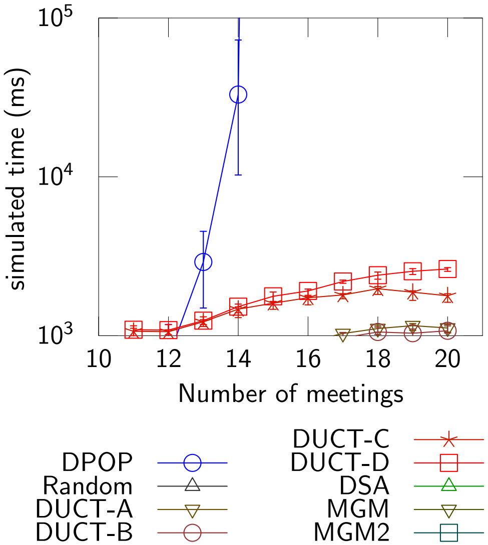


cost
80
100
120
140
160
180
200
220
10
12
14
16
18
20
Number of meetings
DPOP
Random
DUCT-A
DUCT-B
DUCT-C
DUCT-D
DSA
MGM
MGM2

Meeting

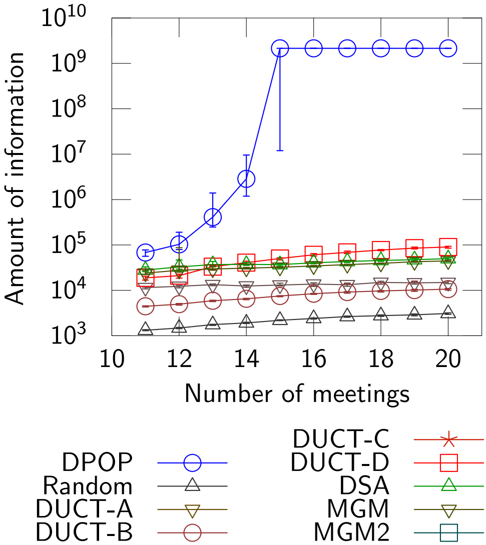

300
250
200
150
100
cost
10
12
14
16
18
20
Number of meetings
DPOP
Random
DUCT-A
DUCT-B
DUCT-C
DUCT-D
DSA
MGM
MGM2

Meeting



10^5
10^4
10^3
simulated time (ms)
10
12
14
16
18
20
Number of meetings
DPOP
Random
DUCT-A
DUCT-B
DUCT-C
DUCT-D
DSA
MGM
MGM2

Meeting



Amount of information
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10
12
14
16
18
20
Number of meetings
DPOP
Random
DUCT-A
DUCT-B
DUCT-C
DUCT-D
DSA
MGM
MGM2