Making Compact-Table Compact

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NordConsNet
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Dynamically Compact Sparse Bit-Sets
Compact Bit-Sets
Dynamically Compact Data-Structures
Sharing Tables
Evaluation
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Compact-Table

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- Propagation algorithm for table constraints [1], [2], [3]

Compact-Table

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- First described in 2016
- Outperforms previously known algorithms

Compact-Table

- Propagation algorithm for table constraints [1], [2], [3]
- Elegant and simple
- First described in 2016
- Outperforms previously known algorithms
- First implemented in OR-tools, now it exists in many solvers

Compact-Table

- Solutions defined by an **explicit table** of tuples

x_0	1	2	1	2	6	7	4	1	1	8	2	1	5	7	3	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Compact-Table

- Solutions defined by an **explicit table** of tuples
- Tuples are numbered from 0 to $n - 1$

x_0	1	2	1	2	6	7	4	1	1	8	2	1	5	7	3	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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Compact-Table

- Solutions defined by an **explicit table** of tuples
- Tuples are numbered from 0 to $n - 1$
- Maintained in a bit-set (array of 64-bit words)

	w_0				w_1				w_2				w_3			
words	**1**	**1**	0	**1**	**1**	0	0	0	**1**	0	**1**	**1**	**1**	0	0	**1**

x_0	**1**	**2**	1	**2**	**6**	7	4	1	**1**	8	**2**	**1**	**5**	7	3	**0**
x_1	**8**	**1**	3	**0**	**7**	4	2	9	**6**	5	**1**	**1**	**0**	5	2	**1**
x_2	**1**	**7**	8	**2**	**4**	9	1	1	**7**	3	**2**	**5**	**1**	9	3	**0**
	0	**1**	2	**3**	**4**	5	6	7	**8**	9	**10**	**11**	**12**	13	14	**15**

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Compact-Table

- Solutions defined by an **explicit table** of tuples
- Tuples are numbered from 0 to $n-1$
- Maintained in a bit-set (array of 64-bit words)
- The i-th bit is set iff the i-th tuple is still valid

	w_0				w_1				w_2				w_3			
words	**1**	**1**	0	**1**	**1**	0	0	0	**1**	0	**1**	**1**	**1**	0	0	**1**

x_0	**1**	**2**	1	**2**	**6**	7	4	1	**1**	8	**2**	**1**	**5**	7	3	**0**
x_1	**8**	**1**	3	**0**	**7**	4	2	9	**6**	5	**1**	**1**	**0**	5	2	**1**
x_2	**1**	**7**	8	**2**	**4**	9	1	1	**7**	3	**2**	**5**	**1**	9	3	**0**
	0	**1**	2	**3**	**4**	5	6	7	**8**	9	**10**	**11**	**12**	13	14	**15**

Support Bit-Sets

- For each variable-value pair $\langle x, v \rangle$

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- Static bit-set masks computed once

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- For each variable-value pair $\langle x, v \rangle$
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- Encode which tuples support the pair

x_0	1	**2**	1	**2**	6	7	**4**	1	1	8	**2**	1	5	7	**3**	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Support Bit-Sets

- For each variable-value pair $\langle x, v \rangle$
- Static bit-set masks computed once
- Encode which tuples support the pair
- The i-th bit set iff tuple nr. i has value v at x's position

x_0	1	**2**	1	**2**	6	7	**4**	1	1	8	**2**	1	5	7	**3**	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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- For each variable-value pair $\langle x, v \rangle$
- Static bit-set masks computed once
- Encode which tuples support the pair
- The i-th bit set iff tuple nr. i has value v at x's position

...

supports$_{\langle x_0, 2\rangle}$

0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0

x_0	1	**2**	1	**2**	6	7	**4**	1	1	8	**2**	1	5	7	**3**	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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Support Bit-Sets

- For each variable-value pair $\langle x, v \rangle$
- Static bit-set masks computed once
- Encode which tuples support the pair
- The i-th bit set iff tuple nr. i has value v at x's position

...

$\text{supports}_{\langle x_0, 2 \rangle}$	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0
$\text{supports}_{\langle x_0, 3 \rangle}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0

x_0	1	**2**	1	**2**	6	7	**4**	1	1	8	**2**	1	5	7	**3**	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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- For each variable-value pair $\langle x, v \rangle$
- Static bit-set masks computed once
- Encode which tuples support the pair
- The i-th bit set iff tuple nr. i has value v at x's position

...

$\texttt{supports}_{\langle x_0, 2\rangle}$	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0
$\texttt{supports}_{\langle x_0, 3\rangle}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0
$\texttt{supports}_{\langle x_0, 4\rangle}$	0	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0	0

...

x_0	1	**2**	1	**2**	6	7	**4**	1	1	8	**2**	1	5	7	**3**	0
x_1	8	1	3	0	7	4	2	9	6	5	1	1	0	5	2	1
x_2	1	7	8	2	4	9	1	1	7	3	2	5	1	9	3	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

A Variable Loses Values

$$\text{dom}(x_0) = \{\not 1, 2, 3, 4, \not 5, \not 6, \not 7, \not 8\}$$

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A Variable Loses Values

$$\mathrm{dom}(x_0) = \{\cancel{1}, 2, 3, 4, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}\}$$

$\texttt{supports}_{\langle x_0, 2\rangle}$			
0 **1** 0 **1**	0 0 0 0	0 0 **1** 0	0 0 0 0

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A Variable Loses Values

$$\mathrm{dom}(x_0) = \{\not 1, 2, 3, 4, \not 5, \not 6, \not 7, \not 8\}$$

$\texttt{supports}_{\langle x_0,2\rangle}$	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0
$\vee$																
$\texttt{supports}_{\langle x_0,3\rangle}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0

A Variable Loses Values

$$\text{dom}(x_0) = \{\cancel{1}, 2, 3, 4, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}\}$$

$\texttt{supports}_{\langle x_0, 2\rangle}$	0 **1** 0 **1**	0 0 0 0	0 0 **1** 0	0 0 0 0
		$\vee$		
$\texttt{supports}_{\langle x_0, 3\rangle}$	0 0 0 0	0 0 0 0	0 0 0 0	0 0 **1** 0
		$\vee$		
$\texttt{supports}_{\langle x_0, 4\rangle}$	0 0 0 0	0 0 **1** 0	0 0 0 0	0 0 0 0
		=		

A Variable Loses Values

$$\mathrm{dom}(x_0) = \{\not{1}, 2, 3, 4, \not{5}, \not{6}, \not{7}, \not{8}\}$$

$\texttt{supports}_{\langle x_0,2\rangle}$	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0
$\vee$																
$\texttt{supports}_{\langle x_0,3\rangle}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0
$\vee$																
$\texttt{supports}_{\langle x_0,4\rangle}$	0	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0	0
=																
mask	0	**1**	0	**1**	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0

A Variable Loses Values

$$\text{dom}(x_0) = \{\not 1, 2, 3, 4, \not 5, \not 6, \not 7, \not 8\}$$

$\texttt{supports}_{\langle x_0, 2\rangle}$	0 1 0 1	0 0 0 0	0 0 1 0	0 0 0 0
∨				
$\texttt{supports}_{\langle x_0, 3\rangle}$	0 0 0 0	0 0 0 0	0 0 0 0	0 0 1 0
∨				
$\texttt{supports}_{\langle x_0, 4\rangle}$	0 0 0 0	0 0 1 0	0 0 0 0	0 0 0 0
=				
mask	0 1 0 1	0 0 1 0	0 0 1 0	0 0 1 0
&				
words (old)	1 1 0 1	1 0 0 0	1 0 1 1	1 0 0 1
=				

A Variable Loses Values

$$\mathrm{dom}(x_0) = \{\not{1}, 2, 3, 4, \not{5}, \not{6}, \not{7}, \not{8}\}$$

$\mathtt{supports}_{\langle x_0, 2\rangle}$	0 1 0 1	0 0 0 0	0 0 1 0	0 0 0 0
			∨	
$\mathtt{supports}_{\langle x_0, 3\rangle}$	0 0 0 0	0 0 0 0	0 0 0 0	0 0 1 0
			∨	
$\mathtt{supports}_{\langle x_0, 4\rangle}$	0 0 0 0	0 0 1 0	0 0 0 0	0 0 0 0
			=	
mask	0 1 0 1	0 0 1 0	0 0 1 0	0 0 1 0
			&	
words (old)	1 1 0 1	1 0 0 0	1 0 1 1	1 0 0 1
			=	
words (new)	0 1 0 1	0 0 0 0	0 0 1 0	0 0 0 0

A Variable Loses Values

$$\mathrm{dom}(x_0) = \{\not{1}, 2, 3, 4, \not{5}, \not{6}, \not{7}, \not{8}\}$$

$\mathtt{supports}_{\langle x_0, 2\rangle}$	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0
∨																
$\mathtt{supports}_{\langle x_0, 3\rangle}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0
∨																
$\mathtt{supports}_{\langle x_0, 4\rangle}$	0	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0	0
=																
mask	0	**1**	0	**1**	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0
&																
words (old)	**1**	**1**	0	**1**	**1**	0	0	0	**1**	0	**1**	**1**	**1**	0	0	**1**
=																
words (new)	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	0	0	0	0

	0	**1**	2	**3**	4	5	6	7	8	9	**10**	11	12	13	14	15
x_0	1	**2**	1	**2**	6	7	4	1	1	8	**2**	1	5	7	3	0
x_1	8	**1**	3	**0**	7	4	2	9	6	5	**1**	1	0	5	2	1
x_2	1	**7**	8	**2**	4	9	1	1	7	3	**2**	5	1	9	3	0

Filtering Variable Domains

- Filter out values where `words` & $\texttt{supports}_{\langle x,v \rangle} = 0$

Filtering Variable Domains

- Filter out values where $\texttt{words} \ \& \ \texttt{supports}_{\langle x,v \rangle} = 0$

$$\text{dom}(\texttt{x}_1) = \{\texttt{0}, \texttt{1}, \texttt{2}, \ldots\}$$

Filtering Variable Domains

- Filter out values where `words` & $\texttt{supports}_{\langle x,v \rangle} = 0$

$$\text{dom}(\mathtt{x}_1) = \{\mathtt{0}, \mathtt{1}, \mathtt{2}, \ldots\}$$

`words`	0	**1**	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0
								&								
$\texttt{supports}_{\langle x_1,0 \rangle}$	0	0	0	**1**	0	0	0	0	0	0	0	0	**1**	0	0	0
								=								
	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0

Value `0` is kept.

Filtering Variable Domains

- Filter out values where $\texttt{words} \ \& \ \texttt{supports}_{\langle x,v \rangle} = 0$

$$\text{dom}(\texttt{x}_1) = \{\texttt{0}, \texttt{1}, 2, \ldots\}$$

Same for 1.

Filtering Variable Domains

- Filter out values where $\texttt{words} \; \& \; \texttt{supports}_{\langle x,v \rangle} = 0$

$\text{dom}(\texttt{x}_1) = \{\texttt{0}, \texttt{1}, \not{\texttt{2}}, \ldots\}$

words	0	**1**	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0
&																
$\texttt{supports}_{\langle x_1,2 \rangle}$	0	0	0	0	0	0	**1**	0	0	0	0	0	**1**	0	**1**	0
=																
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Value $\texttt{2}$ is removed.

Filtering Variable Domains

- Filter out values where $\texttt{words} \,\&\, \texttt{supports}_{\langle x,v\rangle} = 0$

$\text{dom}(\texttt{x}_1) = \{\texttt{0}, \texttt{1}, \not{\texttt{2}}, \ldots\}$

words	0	**1**	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0
&																
$\texttt{supports}_{\langle x_1,2\rangle}$	0	0	0	0	0	0	**1**	0	0	0	0	0	**1**	0	**1**	0
=																
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Value 2 is removed.

And so on. . .

Sparse Bit-Sets

- Indexing structure tracks emptiness
- Operates on non-empty words only
- Performs well even when non-empty words are **sparse**

words	**1 1** 0 **1**	**1** 0 0 0	**1** 0 **1 1**	**1** 0 0 **1**
index	0	1	2	3

limit $= 4$

Sparse Bit-Sets

Intersection with `mask`:

`mask`	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
`words`	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	**1** 0 0 **1**
`index`	0	1	2	3
`limit`	= 4			

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
				&
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	**1** 0 0 **1**
index	0	1	2	3

limit $= 4$

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	0 0 0 0
index	0	1	2	3

limit $= 3$

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	
index	0	1	2	

limit = 3

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
			&	
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	
index	0	1	2	

limit $= 3$

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** **1** 0 **1**	**1** 0 0 0	0 0 **1** **1**	
index	0	1	2	

limit $= 3$

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
		&		
words	**1** **1** 0 **1**	**1** 0 0 0	0 0 **1** **1**	
index	0	1	2	
limit	= 3			

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** **1** 0 **1**	0 0 0 0	0 0 **1** **1**	
index	0	1	2	

limit $= 3$

Sparse Bit-Sets

`mask`	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

`words`	**1** **1** 0 **1**	0 0 0 0	0 0 **1** **1**	
`index`	0	1	2	

`limit` $= 3$

`index`[2] overwrites (or is swapped with) `index`[1]

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
words	**1** **1** 0 **1**	0 0 0 0	0 0 **1** **1**	
index	0	2		

limit = 2

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
	&			
words	**1** **1** 0 **1**	0 0 0 0	0 0 **1** **1**	
index	0	2		
limit	= 2			

Sparse Bit-Sets

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
words	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
index	0	2		

limit $= 2$

Sparse Bit-Sets

	w_0	w_1	w_2	
words	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
index	0	2		
limit	$= 2$			

- Further operations only consider w_0 and w_2

Sparse Bit-Sets

	w_0	w_1	w_2	
words	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
index	0	2		
limit	= 2			

- Further operations only consider w_0 and w_2
- Trailing solver: undo operations upon backtrack

Sparse Bit-Sets

	w_0	w_1	w_2	
words	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
index	0	2		
limit	= 2			

- Further operations only consider w_0 and w_2
- Trailing solver: undo operations upon backtrack
- Copying solver: make copies of the state

Sparse Bit-Sets

	w_0	w_1	w_2	
`words`	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
`index`	0	2		
`limit`	= 2			

- Further operations only consider w_0 and w_2
- Trailing solver: undo operations upon backtrack
- Copying solver: make copies of the state
- `words` is not compact in memory

Sparse Bit-Sets

	w_0	w_1	w_2	
`words`	**1** 0 0 0	0 0 0 0	0 0 **1** **1**	
`index`	0	2		
`limit`	$= 2$			

- Further operations only consider w_0 and w_2
- Trailing solver: undo operations upon backtrack
- Copying solver: make copies of the state
- `words` is not compact in memory
- **Non-compactness problem for a copying solver**

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Compact Bit-Sets

The operations we just watched:

Compact Bit-Sets

The operations we just watched:

$$
\begin{array}{l}
\textbf{for } i \leftarrow \texttt{limit} - 1 \textbf{ downto } 0 \textbf{ do} \\
\quad \texttt{words}[\texttt{index}[i]] \leftarrow_{\&} \texttt{mask}[\texttt{index}[i]] \\
\quad \textbf{if } \texttt{words}[\texttt{index}[i]] = 0 \textbf{ then} \\
\quad\quad \texttt{index}[i] \leftarrow \texttt{index}[\texttt{limit} - 1] \\
\quad\quad \texttt{limit} \leftarrow \texttt{limit} - 1 \\
\quad \textbf{end} \\
\textbf{end}
\end{array}
$$

Compact Bit-Sets

The operations we just watched:

```
for i ← limit − 1 downto 0 do
    words[index[i]] ←& mask[index[i]]
    if words[index[i]] = 0 then
        index[i] ← index[limit − 1]
        limit ← limit − 1
    end
end
```

Compact implementation:

```
for i ← limit − 1 downto 0 do
    words[i] ←& mask[i]
    if words[i] = 0 then
        index[i] ← index[limit − 1]
        words[i] ← words[limit − 1]
        limit ← limit − 1
    end
end
```

Example

Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	**1** 0 0 **1**
index	0	1	2	3

limit $= 4$

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Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
				&
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	**1** 0 0 **1**
index	0	1	2	3

limit $= 4$

Example
Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	0 0 0 0
index	0	1	2	3

limit = 3

Example
Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	
index	0	1	2	

limit $= 3$

Example

Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
			&	
words	**1** **1** 0 **1**	**1** 0 0 0	**1** 0 **1** **1**	
index	0	1	2	
limit	= 3			

Example

Compact Implementation

mask	1 0 1 0	0 0 1 0	0 1 1 1	0 0 1 0

words	1 1 0 1	1 0 0 0	0 0 1 1	
index	0	1	2	

limit $= 3$

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Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
		&		
words	**1** **1** 0 **1**	**1** 0 0 0	0 0 **1** **1**	
index	0	1	2	

limit $= 3$

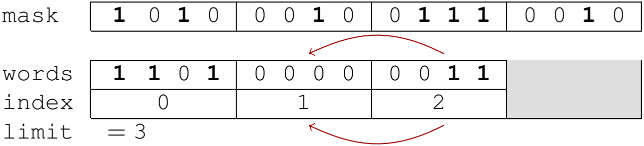
Example

Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
words	**1** **1** 0 **1**	0 0 0 0	0 0 **1** **1**	
index	0	1	2	

limit = 3

Example
Compact Implementation



- index[2] overwrites index[1], and
- words[2] overwrites words[1].

Example
Compact Implementation

mask	1 0 1 0	0 0 1 0	0 1 1 1	0 0 1 0
words	1 1 0 1	0 0 1 1		
index	0	2		

limit $= 2$

Example
Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0
	&			
words	**1** **1** 0 **1**	0 0 **1** **1**		
index	0	2		
limit	= 2			

Example

Compact Implementation

mask	**1** 0 **1** 0	0 0 **1** 0	0 **1** **1** **1**	0 0 **1** 0

words	**1** 0 0 0	0 0 **1** **1**		
index	0	2		

limit $= 2$

Example

Compact Implementation

	w_0	w_2		
words	**1** 0 0 0	0 0 **1** **1**		
index	0	2		
limit	= 2			

- Non-empty words are **contiguous** in memory

Example

Compact Implementation

	w_0	w_2		
`words`	**1** 0 0 0	0 0 **1** **1**		
`index`	0	2		

`limit` $= 2$

- Non-empty words are **contiguous** in memory
- Uses **less indirection** and has better **spatial locality**

Example
Compact Implementation

	w_0	w_2		
`words`	**1** 0 0 0	0 0 **1** **1**		
`index`	0	2		

`limit` = 2

- Non-empty words are **contiguous** in memory
- Uses **less indirection** and has better **spatial locality**
  - `words`[i] $\leftarrow_{\&}$ `mask`[i]
    instead of
    `words`[`index`[i]] $\leftarrow_{\&}$ `mask`[`index`[i]]

Example
Compact Implementation

	w_0	w_2		
words	**1** 0 0 0	0 0 **1** **1**		
index	0	2		
limit	= 2			

- Non-empty words are **contiguous** in memory
- Uses **less indirection** and has better **spatial locality**
  - `words`[i] $\leftarrow_{\&}$ `mask`[i]
    instead of
    `words`[`index`[i]] $\leftarrow_{\&}$ `mask`[`index`[i]]
- Trailing solvers can use the implementation (if elements are swapped)

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Dynamically Compact Data-Structures

- Small tables can be further compacted

Dynamically Compact Data-Structures

- Small tables can be further compacted
- **Specialised bit-sets** used when possible:
  - 16- or 8-bit integers instead of 32 for indexing
  - No indexing for sufficiently small tables

Dynamically Compact Data-Structures

- Small tables can be further compacted
- **Specialised bit-sets** used when possible:
  - 16- or 8-bit integers instead of 32 for indexing
  - No indexing for sufficiently small tables
- Best representation chosen **dynamically** during copying

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Dynamically Compact Data-Structures

- Small tables can be further compacted
- **Specialised bit-sets** used when possible:
  - 16- or 8-bit integers instead of 32 for indexing
  - No indexing for sufficiently small tables
- Best representation chosen **dynamically** during copying
- Most copies created close to the leaves of the search tree, where many words are empty

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Sharing Tables

- Sharing is caring

Sharing Tables

- Sharing is caring
  - ...for memory usage

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Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
  - ...for cache performance

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
  - ...for cache performance
- Tuples and `supports` bit-sets are **shared:**

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
  - ...for cache performance
- Tuples and `supports` bit-sets are **shared:**
  1. Between a propagator and its **copies**

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
  - ...for cache performance
- Tuples and `supports` bit-sets are **shared:**
  1. Between a propagator and its **copies**
  2. Between **different propagators** reasoning on the same set of tuples

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
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- Tuples and `supports` bit-sets are **shared:**
  1. Between a propagator and its **copies**
  2. Between **different propagators** reasoning on the same set of tuples
- `supports` are computed based on the tuples (domain-independent)

Sharing Tables

- Sharing is caring
  - ...for memory usage
  - ...for copying time
  - ...for cache performance
- Tuples and `supports` bit-sets are **shared:**
  1. Between a propagator and its **copies**
  2. Between **different propagators** reasoning on the same set of tuples
- `supports` are computed based on the tuples (domain-independent)
- Sharing `supports` not exploited in the original implementation [1]

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Evaluation Setup

- Standard benchmark set available at http://becool.info.ucl.ac.be/resources/positive-table-constraints-benchmarks
- 1 621 CSP instances (table constraints only), *min-domain* + *min-value* branching strategy
- Solvetime and peak memory usage on top of Gecode

(More detailed description provided on extra slide 22)

Results I

COMPACT	Compact bit-set
COMPACT++	Compact bit-set and compact indexing structure
HYBRID	COMPACT++ and drops indexing for #words ≤ 4

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Results I

COMPACT	Compact bit-set
COMPACT++	Compact bit-set and compact indexing structure
HYBRID	COMPACT++ and drops indexing for #words ≤ 4

Solvetime	COMPACT	COMPACT++	HYBRID
min			
mean			
max			
deviation			

Peak memory	COMPACT	COMPACT++	HYBRID
min			
mean			
max			
deviation			

Results I

COMPACT	Compact bit-set
COMPACT++	Compact bit-set and compact indexing structure
HYBRID	COMPACT++ and drops indexing for #words $\leq$ 4

Solvetime	COMPACT	COMPACT++	HYBRID
min	-67.1%		
mean	-14.4%		
max	0.4%		
deviation	$\pm30.8\%$		

Peak memory	COMPACT	COMPACT++	HYBRID
min	-27.2%		
mean	-4.5%		
max	0.2%		
deviation	$\pm8.5\%$		

Results I

COMPACT Compact bit-set

COMPACT++ Compact bit-set and compact indexing structure

HYBRID COMPACT++ and drops indexing for #words ≤ 4

Solvetime	COMPACT	COMPACT++	HYBRID
min	-67.1%	-66.4%	
mean	-14.4%	-13.7%	
max	0.4%	0.7%	
deviation	$\pm 30.8\%$	$\pm 29.7\%$	

Peak memory	COMPACT	COMPACT++	HYBRID
min	-27.2%	-33.4%	
mean	-4.5%	-6.8%	
max	0.2%	0.0%	
deviation	$\pm 8.5\%$	$\pm 11.4\%$	

Results I

COMPACT	Compact bit-set
COMPACT++	Compact bit-set and compact indexing structure
HYBRID	COMPACT++ and drops indexing for #words ≤ 4

Solvetime	COMPACT	COMPACT++	HYBRID
min	−67.1%	−66.4%	−66.3%
mean	−14.4%	−13.7%	−13.6%
max	0.4%	0.7%	0.9%
deviation	±30.8%	±29.7%	±29.6%

Peak memory	COMPACT	COMPACT++	HYBRID
min	−27.2%	−33.4%	−33.2%
mean	−4.5%	−6.8%	−7.5%
max	0.2%	0.0%	−0.3%
deviation	±8.5%	±11.4%	±11.4%

Results I

COMPACT Compact bit-set

COMPACT++ Compact bit-set and compact indexing structure

HYBRID COMPACT++ and drops indexing for #words ≤ 4

Solvetime	COMPACT	COMPACT++	HYBRID
min	−67.1%	−66.4%	−66.3%
mean	−14.4%	−13.7%	−13.6%
max	0.4%	0.7%	0.9%
deviation	±30.8%	±29.7%	±29.6%

Peak memory	COMPACT	COMPACT++	HYBRID
min	−27.2%	−33.4%	−33.2%
mean	−4.5%	−6.8%	−7.5%
max	0.2%	0.0%	−0.3%
deviation	±8.5%	±11.4%	±11.4%

- Miss rate of D1 cache decreases by $\approx 3\%$ on average

Results II

- **Sharing tables**:
  - Solvetime decreases by 4.6% and memory usage by 58.3% on average
  - Miss rate of D1 cache decreases by $\approx$ 18% on average

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Results II

- **Sharing tables**:
  - Solvetime decreases by 4.6% and memory usage by 58.3% on average
  - Miss rate of D1 cache decreases by $\approx$ 18% on average
- **Previous propagators in Gecode**:
  - Solvetime decreases by 85.7% and memory usage by 45.4% on average
  - Timed out on 85 additional instances

Results II

- **Sharing tables**:
  - Solvetime decreases by 4.6% and memory usage by 58.3% on average
  - Miss rate of D1 cache decreases by $\approx$ 18% on average

- **Previous propagators in Gecode**:
  - Solvetime decreases by 85.7% and memory usage by 45.4% on average
  - Timed out on 85 additional instances

- So called residual supports shown not to be beneficial

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Contributions:

- A **compact implementation** of sparse bit-sets
- Tables are **shared**
- Table constraints in Gecode are about a **magnitude faster** than before and use **half the memory**
- Potential benefit for trailing solvers

Future work:

- Exact variable deltas might speed up propagation
- Re-ordering tuples
- Extensions of compact-table

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References

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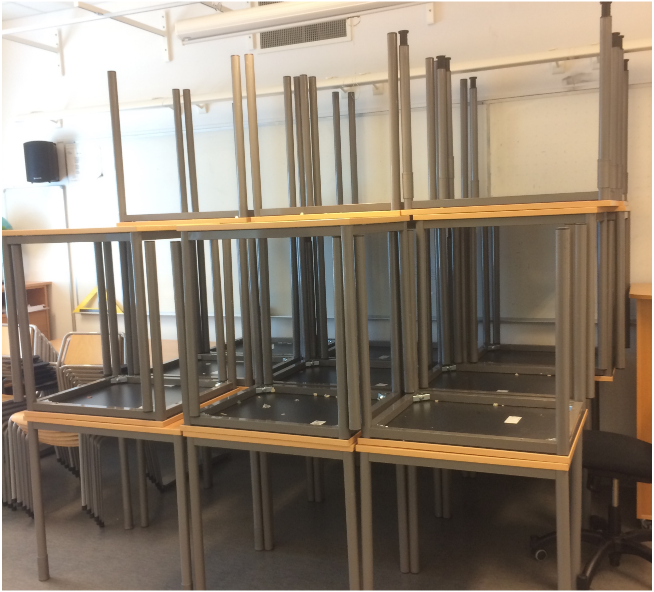
- N. Nethercote, P. J. Stuckey, R. Becket, S. Brand, G. J. Duck and G. Tack, ‘Minizinc: Towards a standard CP modelling language’, in *Proceedings of CP 2007*, 2007, pp. 529–543.

Compact Tables



Detailed Evaluation Setup

- Translated into *MiniZinc* [4] using the tool `xcsp2mzn`, available at https://github.com/CP-Unibo/mzn2feat.
- We skip instances that
  1. cannot be translated to *MiniZinc* due to non-trivial parse errors (117 instances);
  2. require more than 8 GB of RAM (43 instances);
  3. cannot be solved within the time out for the ORIGINAL configuration (170 instances); or
  4. are solved in less than 1 second for the ORIGINAL configuration (1014 instances).

  In total, 277 instances are evaluated.
- Solvetime does not include parsing *FlatZinc*
- Cache analysis uses Cachegrind