Optimal Scheduling in Wireless Networks: Modeling, Solution via Integer Programming, and Recent Developments

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Outline

- Preliminaries
- Maximum link activation: optimization formulation
- Minimum-time link scheduling
- Revisiting maximum link activation
- Rate region: solution characterization
- Incorporating deadlines: an illustration
- Conclusions

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Preliminaries

Generic Network Model

- A set of links (transmitters and receivers)
- Shared radio channel → interference
- Given transmit power (if link active) and path loss coefficients
- Signal-to-interference-and-noise ratio (SINR) threshold

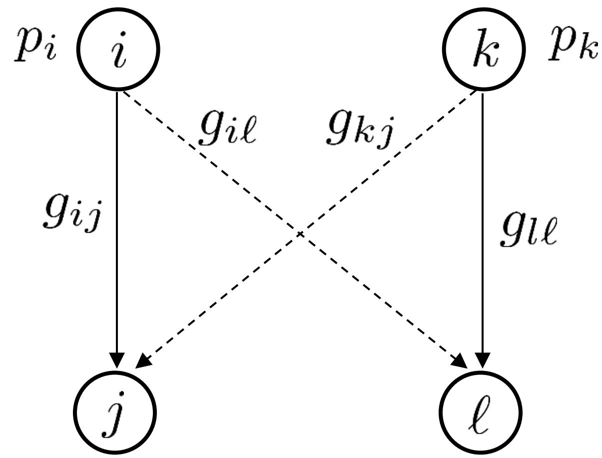

Illustration of two links

$$\text{Success tranmission for } (i,j)\text{: } \mathrm{SINR}_{ij} = \frac{p_i g_{ij}}{p_k g_{kj} + \eta} \geq \gamma$$

Received power

Threshold

Received interference

Noise

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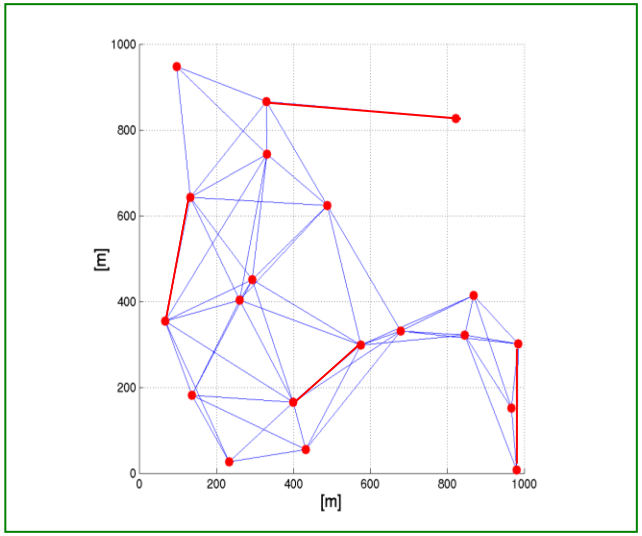
Maximum Link Activation

- What is the maximum number of links that can be in simultaneous (and successful) activation?
- Constraints
  - A node can transmit (receive) to (from) at most one other node
  - SINR threshold
- Intuition: Links that can be activated are “spatially separated”



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Maximum Link Activation: Optimization Formulation

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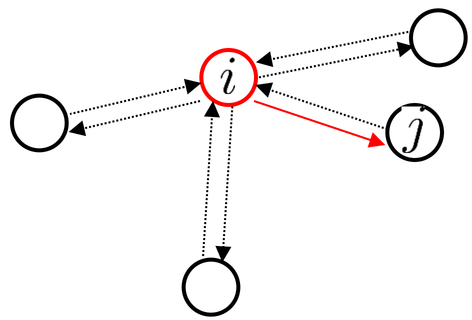
Variables and Constraints

$$x_{ij} = \begin{cases} 1 & \text{if link } (i,j) \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

$$z_i = \begin{cases} 1 & \text{if node } i \text{ transmits,} \\ 0 & \text{otherwise.} \end{cases}$$



By definition: $z_i = \sum_{j \in i^{\rightarrow}} x_{ij}, \ \forall i$

"Simple" constrtaints: $\sum_{j \in i^{\rightarrow}} x_{ij} + \sum_{j \in i^{\leftarrow}} x_{ji} \leq 1, \ \forall i$

SINR threshold: $\frac{p_i g_{ij}}{\sum_{k \neq i} p_k g_{kj} z_k + \eta} \geq \gamma x_{ij}, \ \forall (i,j)$

⇕

$p_i g_{ij} + M(1 - x_{ij}) \geq \gamma (\sum_{k \neq i} p_k g_{kj} z_k + \eta), \ \forall (i,j)$

If M is big enough

Putting the Pieces Together …

$$
\begin{aligned}
\max \quad & \sum_{(i,j)} (\alpha_{ij}) x_{ij} \\
\text{s. t.} \quad & z_i = \sum_{j \in i^{\rightarrow}} x_{ij}, \forall i \\
& \sum_{j \in i^{\rightarrow}} x_{ij} + \sum_{j \in i^{\leftarrow}} x_{ji} \leq 1, \forall i \\
& p_i g_{ij} + M(1 - x_{ij}) \geq \gamma\Big(\sum_{k \neq i} p_k g_{kj} z_k + \eta\Big), \forall (i,j) \\
& \mathbf{x}, \mathbf{z} \in \{0, 1\}
\end{aligned}
$$

Off-the-shelf solver(s) can approach the optimum *fairly well*

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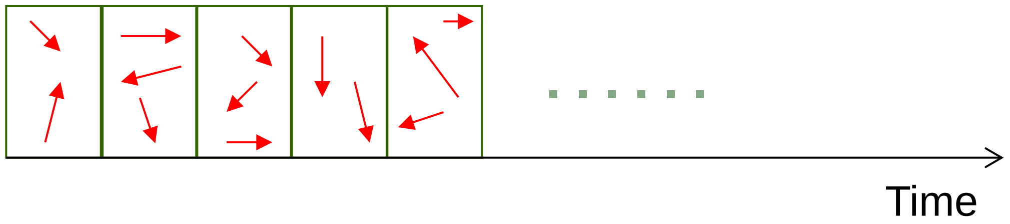
Minimum-Time Link Scheduling

The Concept

- Not all links can be active at the same time → organize transmissions along the time dimension
- Scheduling
  - A subset of links (a solution of the link activation problem) per time slot
  - Minimize the number of time slots necessary to accommodate all links (i.e., each link appears in at least one time slot)



Straightforward Formulation

Define a set of time slots $\{1, 2, \ldots\}$

$$x_{ijt} = \begin{cases} 1 & \text{if link } (i,j) \text{ is active in slot } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{it} = \begin{cases} 1 & \text{if node } i \text{ transmits in slot } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_t = \begin{cases} 1 & \text{if slot } t \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\min \sum_t y_t$$

$$\text{s. t.} \sum_t x_{ijt} \geq 1, \forall (i,j),$$

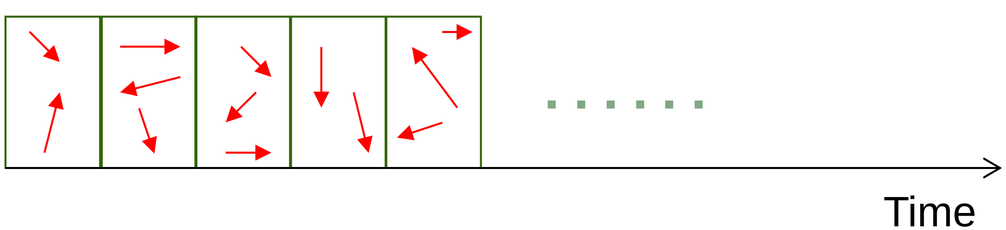
$$x_{ijt} \leq y_t, \forall (i,j), t$$

$$z_{it} = \sum_{j \in i^{\rightarrow}} x_{ijt}, \forall i, t$$

$$\sum_{j \in i^{\rightarrow}} x_{ijt} + \sum_{j \in i^{\leftarrow}} x_{jit} \leq 1, \forall i, t$$

$$p_i g_{ij} + M(1 - x_{ijt}) \geq \gamma (\sum_{k \neq i} p_k g_{kj} z_{kt} + \eta), \forall (i,j), t$$

$$\mathbf{x}, \mathbf{y}, \mathbf{z} \in \{0, 1\}$$



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Sample Results

Nodes	Links	Optimum	Computing time
10	20	17	7h30m
20	134	–	$\gg$10h
30	176	–	$\gg$10h
40	184	–	$\gg$10h
50	296	–	$\gg$10h
60	396	–	$\gg$10h

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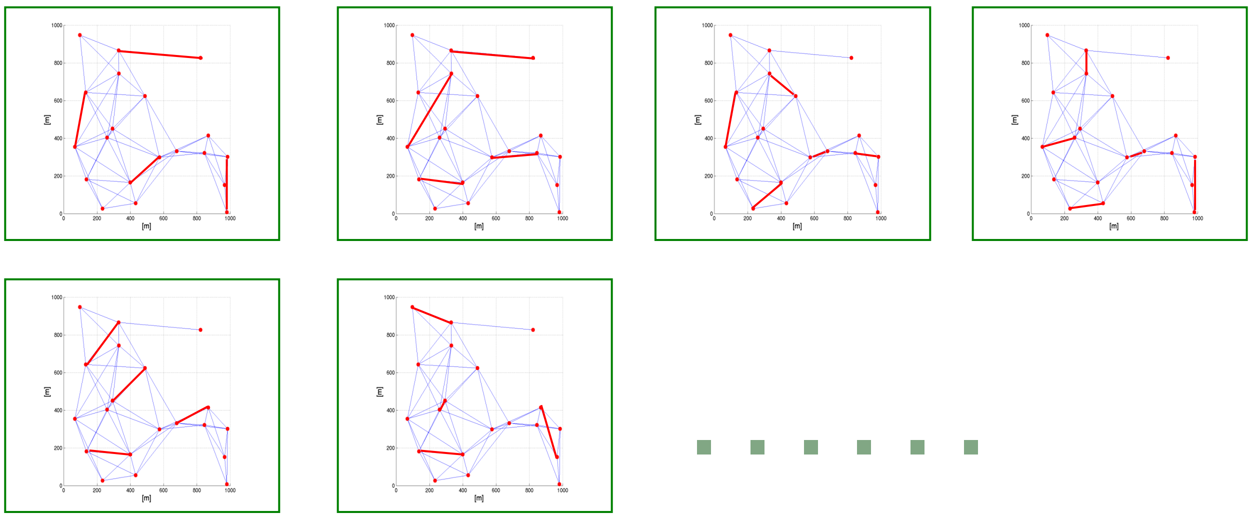
The Notion of Compatible Set

- Compatible set: a feasible link activation set



- If we know all the candidate compatible sets
  - Why care about time slot?
  - Use a minimum number of compatible sets to cover all links (cf. set covering)
- Challenge: exponentially many compatible sets

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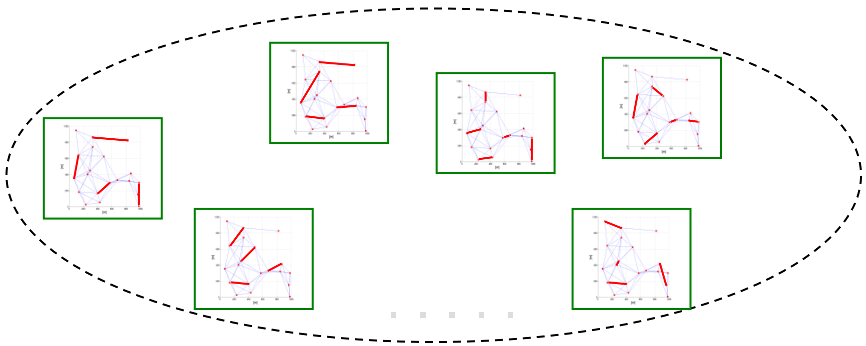
Column Generation

Optimal selection among the generated compatible sets



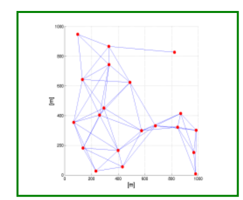
$$\max \sum_{(i,j)} (\alpha_{ij}) x_{ij}$$

$$\text{s. t. } z_i = \sum_{j \in i^{\rightarrow}} x_{ij}, \forall i$$

$$\sum_{j \in i^{\rightarrow}} x_{ij} + \sum_{j \in i^{\leftarrow}} x_{ji} \leq 1, \forall i$$

$$p_i g_{ij} + M(1 - x_{ij}) \geq \gamma(\sum_{k \neq i} p_k g_{kj} z_k + \eta), \forall (i,j)$$

$$\mathbf{x}, \mathbf{z} \in \{0, 1\}$$



Find new compatible sets

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New Results

Nodes	Links	First formulation: Optimum	First formulation: Computing time	New formulation + column generation: Optimum	New formulation + column generation: Computing time
10	20	17	7h30m	17	6s
20	134	–	≫10h	70	4m22s
30	176	–	≫10h	[93, 94]	4m23s
40	184	–	≫10h	[43, 45]	15m47s
50	296	–	≫10h	[84, 85]	1h32m35s
60	396	–	≫10h	[114, 115]	2h52m16s

P. Björklund, P. Värbrand, and D. Yuan. Resource optimization of spatial TDMA in ad hoc networks: a column generation approach. IEEE INFOCOM '03, 2003.

2004-2009: Research on many problem extensions

- Multiple power levels; multiple SINR levels (rate control); multiple packets per link
- Directional antennas
- Routing
- Multicast
- …

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Revisiting Maximum Link Activation

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The (?) Mathematical Formulation

$$\max \quad \sum_{(i,j)} (\alpha_{ij}) x_{ij}$$

$$\text{s. t.} \quad z_i = \sum_{j \in i^{\rightarrow}} x_{ij}, \forall i$$

$$\sum_{j \in i^{\rightarrow}} x_{ij} + \sum_{j \in i^{\leftarrow}} x_{ji} \leq 1, \forall i$$

$$p_i g_{ij} + M(1 - x_{ij}) \geq \gamma(\sum_{k \neq i} p_k g_{kj} z_k + \eta), \forall (i,j)$$

$$\mathbf{x}, \mathbf{z} \in \{0, 1\}$$

- Scheduling amounts to solving link activation multiple times
- What’s “wrong” with the above formulation? Can we reformulate?

A Reformulation

$$p_i g_{ij} + M(1 - x_{ij}) \geq \gamma(\sum_{k \neq i} p_k g_{kj} z_k + \eta)$$

If $x_{ij} = 1$ (link active):

$$p_i g_{ij} \geq \gamma(\sum_{k \neq i} p_k g_{kj} z_k + \eta) \leftrightarrow \sum_{k \neq i} \underbrace{p_k g_{kj}}_{b_{kj}} z_k \leq \underbrace{\frac{p_i g_{ij}}{\gamma} - \eta}_{c_{ij}}$$

$$\sum_{k \neq i} b_{kj} z_k \leq c_{ij}$$

Suppose $b_{1j} + b_{2j} + b_{3j} > c_{ij}$, then at most 2 of the corresponding z-variables can be one $\rightarrow z_1 + z_2 + z_3 \leq 3 - x_{ij}$

In general, if $\sum_{k \in C} b_{kj} > c_{ij}$ for a subset C, then $\sum_{k \in C} z_k \leq |C| - x_{ij}$

- The above is a generalized form of the so called cover inequality
- The SINR constraint can be replaced by (an exponential number of) cover inequalities!

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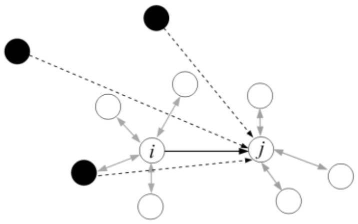
Constraint/Row Generation

Repeatedly add new (and enhanced inequalities), in case of SINR violation

$$\sum_{k \in C} z_k \leq |C| - x_{ij} - \sum_{h \in i^{\rightarrow}} a^C_{ih} x_{ih} - \sum_{h \in i^{\leftarrow}} a^C_{hi} x_{hi}$$



Computing time (s)

Netw.	Links	Convent. model	New algorithm	Netw.	Links	Convent. model	New algorithm
50-1	276	43	26	70-1	588	4161	126
50-2	280	43	13	70-2	630	10071	63
50-3	266	39	7	70-3	610	16333	196
50-4	288	37	3	70-4	560	6682	136
50-5	306	37	5	70-5	644	2751	546
60-1	404	502	55	80-1	732	$\gg$ 10 hours	815
60-2	412	758	24	80-2	826	$\gg$ 10 hours	11252
60-3	408	947	13	80-3	800	26163	4268
60-4	442	318	1	80-4	708	$\gg$ 10 hours	420
60-5	408	112	10	80-5	736	$\gg$ 10 hours	1763

A. Capone, L. Chen, S. Gualandi, and D. Yuan. A new computational approach for maximum link activation in wireless networks under the SINR model. *IEEE Trans. on Wireless Commun.*, 10:1368-1372, 2011.

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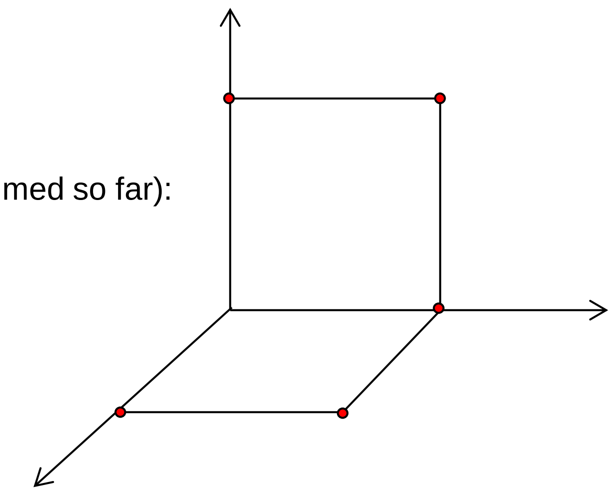
Rate Region: Solution Characterization

Scheduling: New Directions

- Cooperative relaying (many-to-one transmission)
- Interference cancellation
  - Decode (strong) interference first and then remove it
  - Which interfering transmission to decode? In which sequence?
  - SINR or ISNR …
- Scheduling for end-to-end delay: Minimum time span is not optimal
- Scheduling for minimizing age of information
- Rate region

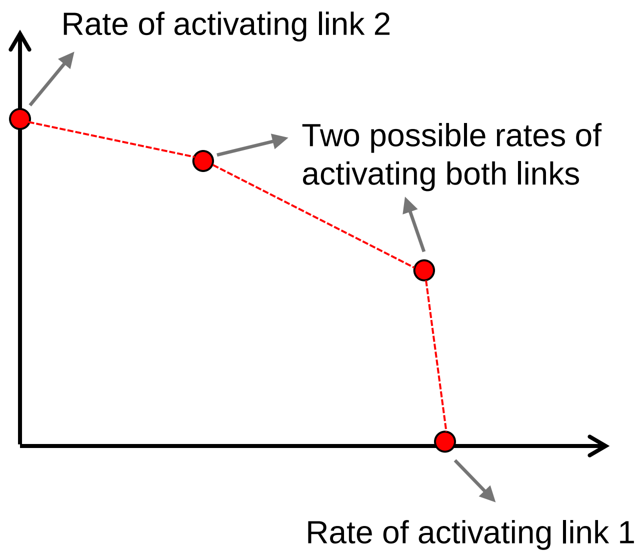
The binary case (assumed so far):

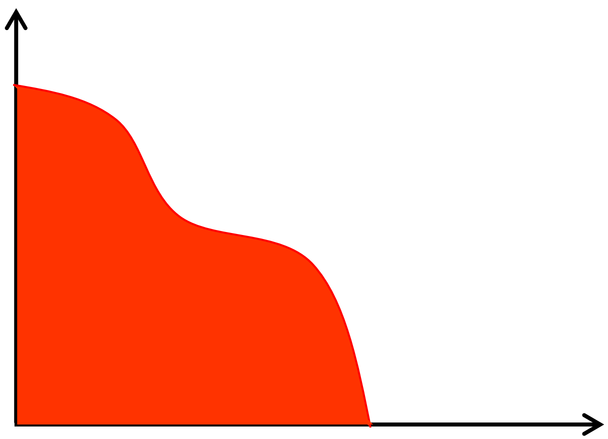


System Assumptions

- An achievable rate region of arbitrary shape; rate = bits/second
- Link data demand: amount of bits to be delivered
- Continuous time

Scheduling: Which subsets of links to activate at what rate and for how long, to minimize the time of meeting the demand?





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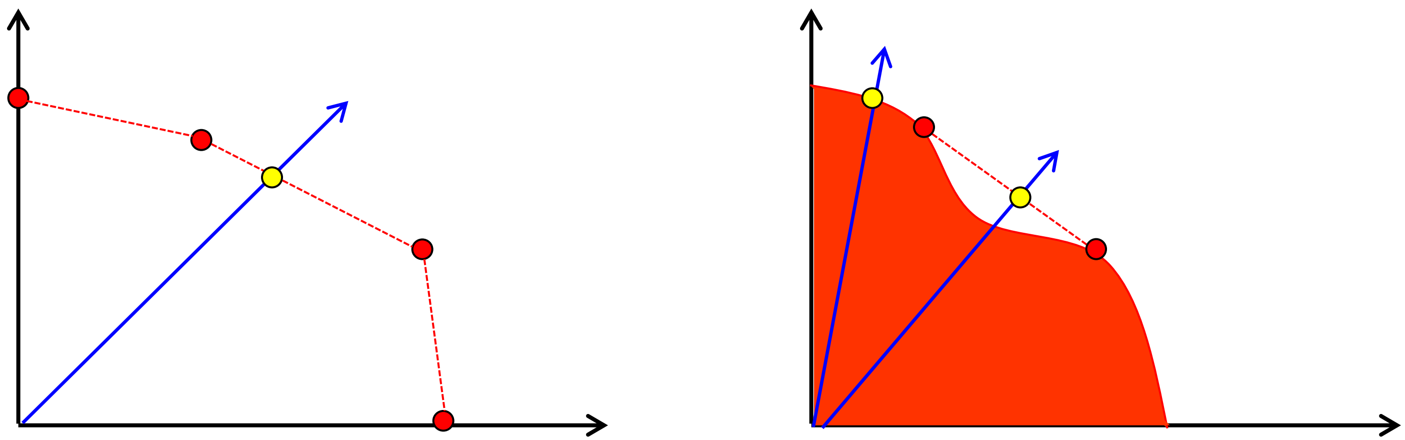
Optimality Characterization

Theorem: The optimum is given by the intersection point of the link demand vector and the boundary of the convex hull of the rate region



Interpretations:

- Minimum-time scheduling = maximum demand scaling
- The intersection point can ne directly implemented: Single rate vector at optimum
- Otherwise: time sharing (= convex combination) of rate vectors

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Incorporating Deadlines: An Illustration

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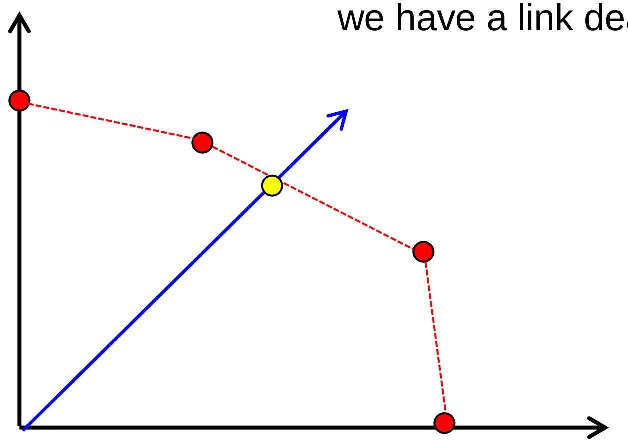
The Basic Concept

- One or multiple links may have deadlines
- The sequence of link activation (and rate vectors) matters
- Can we gain some insight from the previous solution characterization?

How would the optimal point move if we have a link deadline?



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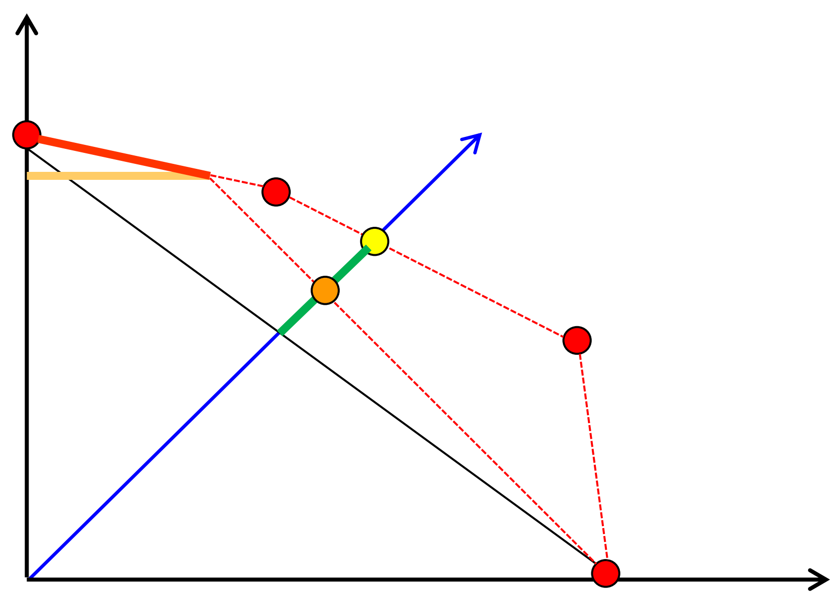
Single Link Deadline:

Suppose one link has a (non-redundant) deadline

- No deadline: The set for feasible schedules correspond to the green segment (why?)
- Deadline → a minimum average rate of the link
- Assuming activations involving the line appear first in the schedule: The average rate of activation containing the link must be on the red segment of the boundary
- Consider the other link, voilà!



Conclusions

- Link activation: a basic resource allocation element in wireless networks
- Scheduling: one step further (and harder)
- Reformulation for scheduling: Isn't it obvious?
- Reformulation for link activation: Isn't it also obvious?
- Extension to rate region and deadline: Well, obvious as well

"In almost anything in life, including in the transmissions in wireless networks, scheduling is a cornerstone for order and performance. Specifically in wireless networking, scheduling has been among the first problems to be studied in depth. It is in fact amazing how the problem unfolds and reveals its multiple facets as we try to formulate it and solve it."

- Anthony Ephremides