Program Transformation and Constraint-based Verification

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CPmeetsCAV 2012, Turunç

Rule-Based Program Transformation - Origins

$$\begin{array}{lccccccl} \text{initial} & P_0 & \longmapsto & \ldots & \longmapsto & P_n & \text{final} \\ & M(P_0) & = & \ldots & = & M(P_n) & \end{array}$$

rule-based
model-preserving
(local) rewriting

An approach to developing correct & efficient programs [Burstall-Darlington 77]

$$\begin{array}{lcccl} \text{'easy-to-prove-correct'} & P_0 & \longmapsto^* & P_n & \text{correct \& efficient} \\ & M(P_0) & = & M(P_n) & \end{array}$$

$$\longmapsto^* \quad \equiv \quad \text{optimization}$$

Separate **correctness** concerns from **efficiency** concerns

$\longmapsto^*$ constructed according to a strategy

Constraint Logic Programming

Programs as sets of rules (clauses) of the form:

$\mathrm{H} \leftarrow c \wedge \mathrm{B}$ (meaning, H holds **if** c is satisfiable in $\mathcal{T}$ and B holds)

Example:

```
ordered([])
ordered([x])
ordered([x1,x2|L]) ← x1 ≤ x2 ∧ ordered([x2|L])
```

$\underbrace{x_1 \leq x_2}_{\text{solver for } \mathcal{T}}$ $\wedge$ $\underbrace{\texttt{ordered([}x_2\texttt{|L])}}_{\text{resolution}}$

Query evaluation:

$d \wedge \mathrm{G} \Longrightarrow^{k}_{\rho} c_1 \wedge \ldots \wedge c_n$ (with $c_1 \wedge \ldots \wedge c_n$ $\mathcal{T}$-satisfiable) (and $\rho = \vartheta_1 \cdot \ldots \cdot \vartheta_k$)

$\Longrightarrow^{1}_{\vartheta}$ **is**

1. $d \wedge \mathrm{G} = d \wedge \mathrm{A} \wedge \mathrm{R}$ current goal
2. $(\mathrm{A} = \mathrm{H})\vartheta$ find unifying head
3. $(d \wedge c \wedge \mathrm{B} \wedge \mathrm{R})\vartheta$ rewrite

Transformation of Constraint Logic Programs

A program as a first order theory: **theory transformation**
(changing the axioms of a theory,
while preserving the model)

Syntax	*Semantics*
logic programs	least Herbrand model
+ negation	perfect model, stable models
+ constraints	least/perfect $\mathcal{D}$-model

Rules

Definition Introduction

$$\longmapsto \quad newp(x) \leftarrow c(x) \wedge p_1(x) \wedge \ldots \wedge p_n(x)$$

Unfolding

$$\boxed{\begin{array}{l} p(x) \leftarrow d_1 \wedge B_1 \\ \quad\vdots \\ p(x) \leftarrow d_n \wedge B_n \end{array}} \quad H \leftarrow c \wedge p(x) \wedge R \quad \longmapsto \quad \begin{array}{l} H \leftarrow c \wedge d_1 \wedge B_1 \wedge R \\ \quad\vdots \\ H \leftarrow c \wedge d_n \wedge B_n \wedge R \end{array}$$

Folding

$$\boxed{p(x) \leftarrow d \wedge B} \quad H \leftarrow c \wedge B \wedge R \quad \overset{c \sqsubseteq d}{\longmapsto} \quad H \leftarrow c \wedge p(x) \wedge R$$

Clause Removal

1. $\begin{array}{l} H \leftarrow c \wedge B \\ H \leftarrow d \end{array} \quad \longmapsto \quad H \leftarrow d$ if $c \sqsubseteq d$ (c entails d)
2. $H \leftarrow c \wedge B \quad \longmapsto \quad \emptyset$ if c is unsatisfiable

Rearrangement, Addition/Deletion, Constraint Rewriting

An Introductory Example

Classical matching: S: L **P** R S = L ++ (**P** ++ R)

P: 2 0

Approximate matching: S: 5 0 4 1 4 3 3 0 3 6 5 1 4

Q

Q is near to **P** with tolerance **K=2**

P_I:

near_match(P,K,S) ← append(L,T,S) ∧ append(Q,R,T) ∧ near(P,K,Q)

near([ ],K,[ ]) ←

near([X|Xs],K,[Y|Ys]) ← X≥Y ∧ X-Y≤K ∧ near(Xs,K,Ys)

near([X|Xs],K,[Y|Ys]) ← X<Y ∧ Y-X≤K ∧ near(Xs,K,Ys)

= element-wise |X-Y| ≤ K

Assume to fix P = [2,0] and K = 2. We introduce the new definition:

snm(S) ← near_match([2,0],2,S) *definitions as patterns*

An Introductory Example

snm(S) ← near_match([2,0],2,S)

An Introductory Example

snm(S) $\leftarrow$ near_match([2,0],2,S)

Unfold near_match([2,0],2,S) (a resolution step):

snm(S) $\leftarrow$ a(L,T,S) $\wedge$ a(Q,R,T) $\wedge$ n([2,0],2,Q)

An Introductory Example

snm(S) ← near_match([2,0],2,S)

Unfold near_match([2,0],2,S) (a resolution step):

snm(S) ← a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

Unfold*

snm([X|S]) ← 0 ≤ X ≤ 2 ∧ a(Q,R,S) ∧ n([0],2,Q)
snm([X|S]) ← 2 < X ≤ 4 ∧ a(Q,R,S) ∧ n([0],2,Q)
snm([X|S]) ← a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

recall :

a([ ],X,X) ←
a([X|Xs],Y,[X|Zs]) ← a(Xs,Y,Zs)

An Introductory Example

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a([X|Xs],Y,[X|Zs]) ← a(Xs,Y,Zs)

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Unfold near_match([2,0],2,S) (a resolution step):

snm(S) ← a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

Unfold*

1. snm([X|S]) ← $0 \le X \le 2$ ∧ a(Q,R,S) ∧ n([0],2,Q)
2. snm([X|S]) ← $2 < X \le 4$ ∧ a(Q,R,S) ∧ n([0],2,Q)

snm([X|S]) ← a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

By merging 1 and 2 and reasoning by cases we can determinize

snm([X|S]) ← $0 \le X \le 4$ ∧ a(Q,R,S) ∧ n([0],2,Q)
snm([X|S]) ← $0 \le X \le 4$ ∧ a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)
snm([X|S]) ← $X < 0$ ∧ a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)
snm([X|S]) ← $X > 4$ ∧ a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

mutually exclusive

An Introductory Example

snm(S) ← near_match([2,0],2,S)

Unfold near_match([2,0],2,S) (a resolution step):

Fold (inverse of Unfold)

snm(S) ← a(L,T,S) ∧ a(Q,R,T) ∧ n([2,0],2,Q)

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mutually exclusive

tupling predicates that share variables

An Introductory Example

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Unfold near_match([2,0],2,S) (a resolution step):

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new1

Definition + Fold

mutually exclusive

An Introductory Example

By Folding, we get

$\mathrm{snm}([X|S]) \leftarrow 0 \leq X \leq 4 \wedge \mathbf{new1}(S)$
$\mathrm{snm}([X|S]) \leftarrow X < 0 \wedge \mathrm{snm}(S)$
$\mathrm{snm}([X|S]) \leftarrow X > 4 \wedge \mathrm{snm}(S)$

where **new1** is defined as follows

$\mathrm{new1}(S) \leftarrow a(Q,R,S) \wedge n([0],2,Q)$
$\mathrm{new1}(S) \leftarrow a(L,T,S) \wedge a(Q,R,T) \wedge n([2,0],2,Q)$

An Introductory Example

By Folding, we get

$snm([X|S]) \leftarrow 0 \leq X \leq 4 \wedge \mathbf{new1}(S)$
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Unfold* + case-split

$new1([X|S]) \leftarrow -2 \leq X \leq 2$
$new1([X|S]) \leftarrow 2 < X \leq 4 \wedge a(Q,R,S) \wedge n([0],2,Q)$
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$new1([X|S]) \leftarrow X < -2 \wedge a(L,T,S) \wedge a(Q,R,T) \wedge n([2,0],2,Q)$ → **snm**
$new1([X|S]) \leftarrow X > 4 \wedge a(L,T,S) \wedge a(Q,R,T) \wedge n([2,0],2,Q)$ → **snm**

Fold (the two boxed bodies of the 2nd and 3rd clauses → the bodies of new1(S))

mutually exclusive

Fold

An Introductory Example

The final program P_F :

$snm([X|S]) \leftarrow 0 \leq X \leq 4 \wedge new1(S)$
$snm([X|S]) \leftarrow X < 0 \wedge snm(S)$
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An Introductory Example

The final program P_F :

snm([X|S]) ← 0 ≤ X ≤ 4 ∧ new1(S)
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Correctness:

For all S

$M(P_I) \models$ near_match([2,0],2,S)

iff

$M(P_F) \models$ snm(S)

where M( ) denotes the least *D-model*

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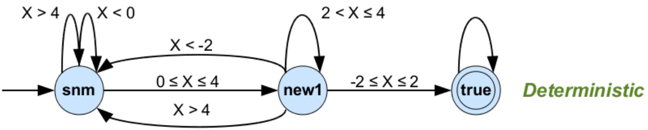

Transformation Strategies

Directed by syntactic features of programs:

- Specializing programs to the context of use (pre-computing)

  snm(S) ← near_match(**[2,0]**,**2**,S)

- Avoiding the computation of unnecessary values

  near_match(P,K,S) ← a(**L**,T,S) ∧ a(Q,R,T) ∧ near(P,K,Q)

- Avoding multiple visits of data structures and repeated computations

  near_match(P,K,S) ← a(L,**T**,S) ∧ a(**Q**,R,**T**) ∧ near(P,K,**Q**)

- Reducing nondeterminism (avoid multiple matchings per call-pattern)

  snm([X|S]) ← **0≤X≤4** ∧ a(Q,R,S) ∧ n([0],2,Q)
  snm([X|S]) ← **0≤X≤4** ∧ a(L,T,S) ∧ a(Q,R,T) ∧ near([2,0],2,Q)

  ⇓

  snm([X|S]) ← **0≤X≤4** ∧ new1(S)

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  $\Downarrow$

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  ⇓

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Rule-Based Program Transformation - More

P_0	$\longmapsto$	$\ldots$	$\longmapsto$	P_n	*What do we preserve?*
$M(P_0)$	$=$	$\ldots$	$=$	$M(P_n)$	a model
$A \in M(P_0)$	iff	$\ldots$	iff	$A \in M(P_n)$	selected predicates
$M(P_0) \models \varphi$	iff	$\ldots$	iff	$M(P_n) \models \varphi$	a class of formulas

$\longmapsto^* \equiv$ deduction

Depending on the choice of the set of **rules** and the **transformation strategy**

Applications

Theorem Proving

[Kott,P,P,Roychoudhury,Seki]

$T \cup \{p \leftarrow \varphi\} \longmapsto^* S \cup \{p \leftarrow\}$

Program Verification

[Albert,Gallagher,Puebla]

$[\![P]\!] \cup \{p \leftarrow \varphi\} \longmapsto^* Q \cup \{p \leftarrow\}$

where $[\![P]\!]$ is an encoding of the program semantics (e.g., an interpreter)

Program Synthesis

[Darlington,Deville,Flener,Hogger,Lau,Manna,Waldinger]

$T \cup \{p(x) \leftarrow \varphi(x)\} \longmapsto^* P$

s.t. $T \models \varphi(a)$ iff $p(a) \in M(P)$

Improving Infinite-state Systems Model Checking

Specialization-based Model Checking

Program Specialization

$\mathcal{P} : I_1 \times I_2 \longrightarrow O$

By partial evaluation

$\mathcal{P}_1 : I_2 \longrightarrow O$ A faster residual program

Take advantage of static knowledge

Specialization-based Symbolic Model Checking

$\mathcal{CTL} : TS \times \varphi \times Parameters \longrightarrow$ yes/no

By partial evaluation

$\mathcal{MC}^{\varphi}_{TS} : Parameters \longrightarrow$ yes/no An ad-hoc model checker

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Specialization Strategy

Input: P and a clause δ_0: $p_{sp}(x) \leftarrow c(x) \wedge p(x)$

Output: *SpecP* s.t. $p_{sp}(z) \in M(P \cup \{\delta_0\})$ iff $p_{sp}(z) \in M(SpecP)$

$SpecP := \emptyset$;

$Defs := \{\delta_0\}$;

while $\exists\, \delta \in Defs$ **do**

$\Gamma := \mathit{Unfold}\ \delta$

$\Delta := \mathit{Simplify}\ \Gamma$

$(\Phi, NewDefs) := \mathit{Generalize\&Fold}\ \Delta$

$Defs := (Defs - \{\delta\}) \cup NewDefs$

$SpecP := SpecP \cup \Phi$

od

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$SpecP := \emptyset$;

$Defs := \{\delta_0\}$;

while $\exists\, \delta \in Defs$ **do**

$\Gamma := Unfold\ \delta$ (Propagate Context)

$\Delta := Simplify\ \Gamma$

$(\Phi, NewDefs) := Generalize\&Fold\ \Delta$ (Apply Induction)

$Defs := (Defs - \{\delta\}) \cup NewDefs$

$SpecP := SpecP \cup \Phi$

od

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od

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- **Automated**
- **Terminating**

Application to Backward Reachability

Infinite-State System : $\langle Var, I, T, U \rangle$ (constraints on $\mathcal{Z}_{lin}$)

Given a set S of states, $\text{PRE}(T,S) = \{X \mid \exists X' \in S \;\; s.t. \;\; T(X,X')\}$

Goal: Check that $\text{PRE}^{\omega}(T,U) \cap I = \emptyset$ (undecidable)

Problem: The computation of $\text{PRE}^{\omega}(T,U)$ may not terminate

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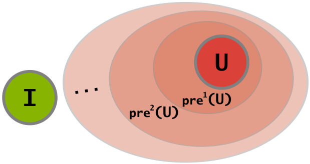


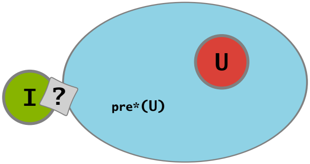
Application to Backward Reachability

Infinite-State System : $\langle Var, I, T, U \rangle$ (constraints on $\mathcal{Z}_{lin}$)

Given a set S of states, $\text{PRE}(T,S) = \{X \mid \exists X' \in S \;\; s.t. \;\; T(X,X')\}$

Goal: Check that $\text{PRE}^{\omega}(T,U) \cap I = \emptyset$ (undecidable)

Problem: The computation of $\text{PRE}^{\omega}(T,U)$ may not terminate



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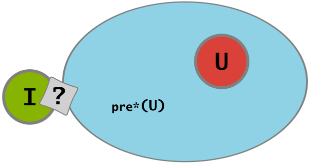

knowledge of the ***target*** (I)
is not taken into account
in the construction of $\mathrm{PRE}^{\omega}(T,U)$

Encoding (Backward) Reachability in CLP

$$Sys = \langle Var, I, T, U \rangle \quad \text{where} \quad \begin{array}{rl} I: & init_1(X) \vee \ldots \vee init_k(X) \\ T: & t_1(X, X') \vee \ldots \vee t_m(X, X') \\ U: & u_1(X) \vee \ldots \vee u_n(X) \end{array}$$

$$\begin{array}{rrcl} I_1: & not\_safe & \leftarrow & init_1(X) \wedge bwReach(X) \\ & & \vdots & \\ I_k: & not\_safe & \leftarrow & init_k(X) \wedge bwReach(X) \\ T_1: & bwReach(X) & \leftarrow & t_1(X, X') \wedge bwReach(X') \\ & & \vdots & \\ T_m: & bwReach(X) & \leftarrow & t_m(X, X') \wedge bwReach(X') \\ U_1: & bwReach(X) & \leftarrow & u_1(X) \\ & & \vdots & \\ U_n: & bwReach(X) & \leftarrow & u_n(X) \end{array}$$

see also [Fribourg 97, Delzanno-Podelski 99]

full CTL encoded similarly [e.g. LOPSTR10]

Source-to-Source Specialization

System S $\Downarrow$ $P_S \in \mathrm{CLP}(\mathcal{Z})$ $\overset{\textit{specialize}}{\Longrightarrow}$ $\mathit{SpecP}_S \in \mathrm{CLP}(\mathcal{Z})$ $\Uparrow$ Specialized System SpecS

Input: $S = \langle \mathit{Var}, I, T, U \rangle$

BACKWARD we specialize w.r.t. the *Initial States*

$$\mathrm{PRE}^{\omega}(T, U) \cap I = \emptyset \quad \textbf{iff} \quad \mathrm{PRE}^{\omega}_{T,I}(U) = \emptyset$$

Output: $\mathit{SpecS} = \langle \mathit{SpecVar}, \mathit{SpecI}, \mathit{SpecT}, \mathit{SpecU} \rangle$

The standard operator PRE^{ω} behaves on *SpecS* as $\mathrm{PRE}^{\omega}_{T,I}$

Source-to-Source Specialization

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Source-to-Source Specialization

$$
\begin{array}{ccc}
\text{System } S & & \text{Specialized System } SpecS \\
\Downarrow & & \Uparrow \\
P_S \in \mathrm{CLP}(\mathcal{Z}) & \overset{specialize}{\Longrightarrow} & SpecP_S \in \mathrm{CLP}(\mathcal{Z})
\end{array}
$$

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	default		F	
EXAMPLES	*Sys*	*SpSys*	*Sys*	*SpSys*
Bakery2	0.03	0.05	0.06	0.04
Bakery3	0.70	0.25	∞	3.68
MutAst	1.46	0.37	0.22	0.59
Peterson	56.49	0.10	∞	13.48
Ticket	∞	0.03	0.02	0.19
Berkeley RISC	0.01	0.04	0.01	0.02
DEC Firefly	0.01	0.02	0.01	0.07
IEEE Futurebus	0.26	0.68	∞	∞
Illinois Cache Coherence	0.01	0.03	∞	0.07
Barber	0.62	0.21	∞	0.08
CSM	56.39	7.69	∞	125.32
Consistency	∞	0.11	∞	324.14
Insertion Sort	0.03	0.06	0.18	0.02
Selection Sort	∞	0.21	∞	0.33
Reset Petri Net	∞	0.02	∞	0.01
Train	42.24	59.21	∞	0.46
No. of verified properties	12	16	6	15
	BACKWARD		FORWARD	

Timings : *Sys* $=$ **fixpoint only**
SpSys $=$ **specialization** $+$ **fixpoint**

fixpoint computed using ALV [Bultan et al. 09], based on the Omega library

'$\perp$' = 'Unable to verify' and '∞' = 'Timeout' (10 minutes)

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BACKWARD FORWARD

- Specialization *improves precision*
- Overall, it *does not deteriorate verification time*
- Applicable in both *forward and backward* analyses

Specialization-based Software Model Checking

$$
\begin{array}{lcl}
a & ::= & n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 \times a_2 \\
b & ::= & \textbf{true} \mid \textbf{false} \mid a_1 \ op \ a_2 \mid !\, b \mid b_1 \ \&\& \ b_2 \mid b_1 \ \| \ b_2 \\
t & ::= & * \mid b \\
c & ::= & skip \mid x = a \mid c_1 ; c_2 \mid \textbf{if } t \textbf{ then } c_1 \textbf{ else } c_2 \mid \textbf{while } t \textbf{ do } c \textbf{ od}
\end{array}
$$

CLP interpreter for the operational semantics of SIMP

```
t(s(skip,S), E)
t(s(asgn(var(X),A),E),s(skip,E1))  ←  aeval(A,S,V), update(var(X),V,S,E1)
t(s(comp(C0,C1),S), s(C1,S1))  ←  t(s(C0,S),S1)
t(s(comp(C0,C1),S), s(comp(C0',C1),S'))  ←  t(s(C0,S), s(C0',S'))
t(s(ite(B,C0,_),S), s(C0,S))  ←  beval(B,S)
t(s(ite(B,_,C1),S), s(C1,S))  ←  beval(not(B),S)
t(s(ite(ndc,S1,_),E),s(S1,E))
t(s(ite(ndc,_,S2),E),s(S3,E))
t(s(while(B,C),S),s(ite(B,comp(C,while(B,C)),skip),S))
```

see also [Peralta,Gallagher SAS98+LOPSTR99]

Summarizing

Pros/Cons:

- improvement of termination but increase in size
- φ-preserving and terminating
- independent of the verification tool/independent of the verification tool

Features:

- control on **termination/precision** (*wqos* and *generalizations*)
- $\mathcal{Z}$ **hard** to solve: *precise relaxation* to $\mathcal{R}$
- **logics**: *CTL** and *ω-regular languages*
- residual program **size**: *controlling polyvariance*

Reasoning on Data Structures + Constraints

Context & Related Work

- Transformation and Theorem Proving:
  - equivalence proofs by unfold/fold [Kott-82, PP-99, Roychoudhury et al-99]
  - first order theorem proving by unfold/fold [PP-00]

- Deforestation [Wadler-90] used for quantifier elimination
  A technique for the elimination of intermediate data structures

- No existential variables entails No intermediate data structures
  Existential variables occur in the body of a clause and not in the head

  E.g., X existential in $p \leftarrow q(X) \wedge r(X)$

  Since $\forall X(\, p \leftarrow q(X) \wedge r(X)\,) \equiv p \leftarrow \exists X(q(X) \wedge r(X))$

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Proving Properties of LR programs

LR programs:

- linear constraints on $\mathcal{R}/\mathcal{Q}$
- linear recursion + negation
- no existential variables

Example:

P: $member(X,[Y|L]) \leftarrow X = Y$
$member(X,[Y|L]) \leftarrow member(X,L)$

$\varphi: \ \forall L \exists U \forall X\, (member(X,L) \rightarrow X \leq U)$

Checking $M(P) \models \varphi$, for any LR-program P and closed formula φ, is undecidable (Peano arithmetic can be encoded)

Quantifier elimination cannot be algorithmic

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QE for CLP(Rlin) via Program Transformation

Step 1. Using a variant of Lloyd-Topor transformation,
we obtain a clause form CF for φ s.t. $M(P) \models \varphi$ iff $M(P \cup CF) \models f$

$$\forall L \quad \exists U \quad \forall Y\ (\text{member}(Y, L) \rightarrow Y \leq U)$$

$$p \leftarrow \underbrace{\neg\ \exists L\ \underbrace{\neg\ \exists U\ \neg\ \underbrace{\exists Y(\text{member}(Y, L) \wedge Y > U)}_{s}}_{r}}_{q}$$

CF:

D_4: $p \leftarrow \neg q$
D_3: $q \leftarrow list(L) \wedge \neg r(L)$
D_2: $r(L) \leftarrow list(L) \wedge \neg s(L, U)$
D_1: $s(L, U) \leftarrow Y > U \wedge list(L) \wedge member(Y, U)$

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- non LR-clauses (existential variables, nonlinear)

Resolution does not terminate on the query ***p***

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D_1: $\boldsymbol{s}(L, U) \leftarrow Y > U \wedge \mathit{list}(L) \wedge \mathit{member}(Y, U)$

- stratified
- non LR-clauses (existential variables, nonlinear)

Resolution does not terminate on the query $\boldsymbol{p}$

Quantifiers can be eliminated by deriving LR-programs

QE for CLP(Rlin) via Program Transformation - Cont.

Step 2. Apply a strategy that combines unfold/fold transformations and constraint reasoning on $\mathcal{R}_{lin}$

Goal: from $P \cup CF$ derive a propositional program Prop s.t. $M(P \cup CF) \models f$ iff $M(Prop) \models f$

Prop:

$$p \leftarrow \neg\, q$$
$$q \leftarrow newp$$
$$newp \leftarrow newp$$

As a consequence, $M(P) \models \varphi$ iff $M(Prop) \models p$

The transformation from $P \cup CF$ to Prop consists in eliminating all existential variables from CF

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The Transformational Proof Method

D_1: $s(L,U) \leftarrow X > U \wedge list(L) \wedge member(X,L)$

The Transformational Proof Method

D_1: s(L,U) $\leftarrow$ X > U $\wedge$ list(L) $\wedge$ member(X,L)

Unfold: s([X|T],U) $\leftarrow$ X > U $\wedge$ list(T)

s([Y|T],U) $\leftarrow$ X > U $\wedge$ list(T) $\wedge$ member (X,T)

The Transformational Proof Method

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Fold: $s([X|T],U) \leftarrow X > U \wedge list(T)$

$s([X|T],U) \leftarrow s(T,U)$ LR-clauses

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D_4: $p \leftarrow \neg q$

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D_2: $r(L) \leftarrow list(L) \wedge \neg s(L, U)$

$\mathbf{D_1}$: $s([X|T],U) \leftarrow X > U \wedge list(T)$

$s([X|T],U) \leftarrow s(T,U)$

The Transformational Proof Method

D_2: $r(L) \leftarrow list(L) \wedge \neg s(L,U)$

The Transformational Proof Method

D_2: $r(L) \leftarrow \boxed{list(L)} \wedge \boxed{\neg\, s(L,U)}$

Unfold: $r([\,])$
$r([X|Xs]) \leftarrow X \leq U \wedge list(Xs) \wedge \neg\, s(Xs,U)$

The Transformational Proof Method

D_2: $r(L) \leftarrow list(L) \wedge \neg s(L,U)$

Unfold: $r([\,])$

$r([X|Xs]) \leftarrow X \leq U \wedge list(Xs) \wedge \neg s(Xs,U)$ BAD folding

The Transformational Proof Method

D₂: $r(L) \leftarrow list(L) \land \neg s(L,U)$

Unfold: $r([\,])$
$r([X|Xs]) \leftarrow X \leq U \land list(Xs) \land \neg s(Xs,U)$

Define: $new_1(X,L) \leftarrow X \leq U \land list(L) \land \neg s(L,U)$

The Transformational Proof Method

D_2: $r(L) \leftarrow list(L) \wedge \neg s(L,U)$

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LR-clauses

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LR-clauses

Unfold: $new_1(X,[\,]) \leftarrow$

$new_1(X,[Y|Ys]) \leftarrow X \leq U \land Y \leq U \land list(Ys) \land \neg s(Ys,U)$

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$\equiv (Y<X \wedge X \leq U) \vee (X \leq Y \wedge Y \leq U)$ (linear order)

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Replace: $new_1(X,[Y|Ys]) \leftarrow Y < X \wedge X \leq U \wedge list(Ys) \wedge \neg s(Ys,U)$

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$new_1(X,[Y|Ys]) \leftarrow X \leq Y \wedge \boxed{Y \leq U \wedge list(Ys) \wedge \neg s(Ys,U)}$

Fold: $new_1(X,[\,]) \leftarrow$

$new_1(X,[Y|Ys]) \leftarrow Y<X \wedge \boxed{new_1(X,Ys)}$

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$\equiv (Y<X \wedge X \leq U) \vee (X \leq Y \wedge Y \leq U)$ (linear order)

Replace:
$new_1(X,[Y|Ys]) \leftarrow Y<X \wedge X \leq U \wedge list(Ys) \wedge \neg s(Ys,U)$
$new_1(X,[Y|Ys]) \leftarrow X \leq Y \wedge Y \leq U \wedge list(Ys) \wedge \neg s(Ys,U)$

Fold:
$new_1(X,[\,]) \leftarrow$
$new_1(X,[Y|Ys]) \leftarrow Y<X \wedge \boxed{new_1(X,Ys)}$
$new_1(X,[Y|Ys]) \leftarrow X \leq Y \wedge \boxed{new_1(Y,Ys)}$

LR-clauses

The Transformational Proof Method

D_4: $\boldsymbol{p} \leftarrow \neg\, \boldsymbol{q}$

D_3: $\boldsymbol{q} \leftarrow list(L) \wedge \neg\, \boldsymbol{r}(L)$

D_2: $r([\,]) \leftarrow$
$r([X|Xs]) \leftarrow new_1(X,Xs)$

$new_1(X,[\,]) \leftarrow$
$new_1(X,[Y|Ys]) \leftarrow Y<X \wedge new_1(X,Ys)$
$new_1(X,[Y|Ys]) \leftarrow X \leq Y \wedge new_1(Y,Ys)$

D_1: $s([X|T],U) \leftarrow X > U \wedge list(T)$
$s([X|T],U) \leftarrow s(T,U)$

The Transformational Proof Method

D₄: $p \leftarrow \neg\, q$

D₃: $q \leftarrow new_2$

$new_2 \leftarrow new_2$

D₂: $r([\,]) \leftarrow$
$r([X|Xs]) \leftarrow new_1(X,Xs)$

$new_1(X,[\,]) \leftarrow$
$new_1(X,[Y|Ys]) \leftarrow Y<X \wedge new_1(X,Ys)$
$new_1(X,[Y|Ys]) \leftarrow X \leq Y \wedge new_1(Y,Ys)$

D₁: $s([X|T],U) \leftarrow X > U \wedge list(T)$
$s([X|T],U) \leftarrow s(T,U)$

Using:

- domain axioms
- quantifier elimination on constraints

The Transformational Proof Method

D$_4$: $p \leftarrow \neg\, q$

D$_3$: $q \leftarrow new_2$

$new_2 \leftarrow new_2$

D$_2$: $r([\,]) \leftarrow$
$r([X|Xs]) \leftarrow new_1(X,Xs)$

$new_1(X,[\,]) \leftarrow$
$new_1(X,[Y|Ys]) \leftarrow Y{<}X \wedge new_1(X,Ys)$
$new_1(X,[Y|Ys]) \leftarrow X{\leq}Y \wedge new_1(Y,Ys)$

D$_1$: $s([X|T],U) \leftarrow X > U \wedge list(T)$
$s([X|T],U) \leftarrow s(T,U)$

Using:

- domain axioms
- quantifier elimination on constraints

Since $M(Prop) \models p$, we conclude $M(P) \models \varphi$

Some Observations

Summary:

- a heuristic
- a decision procedure for wS1S

Challenge:

property driven, automatic inference of needed axioms

Optimizing Test-case Generation

Filter Promotion

N-Queens

$$
\begin{array}{lll}
\text{queens}(X,C) & \leftarrow & \text{generate}(N,C) \wedge \text{check}(C) \\
\text{generate}(N,C) & \leftarrow & \text{permutation}(N,C) \\
\text{check}([\,]) & \leftarrow & \\
\text{check}([Q|Qs]) & \leftarrow & \neg\,\text{attack}(Q,Qs) \wedge \\
 & & \text{check}(Qs)
\end{array}
$$

Filter Promotion

N-Queens

queens(X,C) ← generate(N,C) ∧ check(C)

generate(N,C) ← permutation(N,C)

check([ ]) ←
check([Q|Qs]) ← ¬ attack(Q,Qs) ∧
check(Qs)

Q			
		Q	
	Q		
			Q

X

Filter Promotion

N-Queens

$$\text{queens}(X,C) \leftarrow \text{generate}(N,C) \wedge \text{check}(C)$$

$$\text{generate}(N,C) \leftarrow \text{permutation}(N,C)$$

$$\text{check}([\,]) \leftarrow$$
$$\text{check}([Q|Qs]) \leftarrow \neg\, \text{attack}(Q,Qs) \wedge \text{check}(Qs)$$

		Q	
Q			
			Q
	Q		

✓

Filter Promotion

N-Queens

queens(X,C) ← generate(N,C) ∧ check(C)

generate(N,C) ← permutation(N,C)

check([ ]) ←
check([Q|Qs]) ← ¬ attack(Q,Qs) ∧
check(Qs)

		Q	
Q			
			Q
	Q		

✓

queens(X,C) ← queens(N,[ ],C)

queens(0,Qs,Qs) ←
queens(N,SafeQs,Qs) ← place(SafeQs,**Q**) ∧
¬ attack(**Q**,SafeQs) ∧ ⟸ promoted
M>0 ∧ M is N-1 ∧
queens(M,[**Q**|SafeQs],Qs)

Filter Promotion

N-Queens

```
queens(X,C)     ← generate(N,C) ∧ check(C)
generate(N,C) ← permutation(N,C)
check([ ])        ←
check([Q|Qs])  ← ¬ attack(Q,Qs) ∧
                        check(Qs)
```

		Q	
Q			
			Q
	Q		

✓

```
queens(X,C)                  ← queens(N,[ ],C)
queens(0,Qs,Qs)            ←
queens(N,SafeQs,Qs) ← place(SafeQs,Q) ∧
                                   ¬ attack(Q,SafeQs) ∧
                                   M>0 ∧ M is N-1 ∧
                                   queens(M,[Q|SafeQs],Qs)
```

		Q	
Q			
			Q

Filter Promotion

N-Queens

queens(X,C) ← generate(N,C) ∧ check(C)

generate(N,C) ← permutation(N,C)

check([ ]) ←
check([Q|Qs]) ← ¬ attack(Q,Qs) ∧
check(Qs)

		Q	
Q			
			Q
	Q		

✓

queens(X,C) ← queens(N,[ ],C)

queens(0,Qs,Qs) ←
queens(N,SafeQs,Qs) ← place(SafeQs,**Q**) ∧
¬ attack(**Q**,SafeQs) ∧
M>0 ∧ M is N-1 ∧
queens(M,[**Q**|SafeQs],Qs)

		Q	
Q			
			Q
Q			

✗

Filter Promotion

N-Queens

```
queens(X,C)      ← generate(N,C) ∧ check(C)
generate(N,C)    ← permutation(N,C)
check([ ])       ←
check([Q|Qs])    ← ¬ attack(Q,Qs) ∧
                   check(Qs)
```

		Q	
Q			
			Q
	Q		

✓

```
queens(X,C)            ← queens(N,[ ],C)
queens(0,Qs,Qs)        ←
queens(N,SafeQs,Qs)    ← place(SafeQs,Q) ∧
                         ¬ attack(Q,SafeQs) ∧
                         M>0 ∧ M is N-1 ∧
                         queens(M,[Q|SafeQs],Qs)
```

		Q	
Q			
			Q
	Q		

✓

Filter Promotion

N-Queens

```
queens(X,C)      ← generate(N,C) ∧ check(C)
generate(N,C)    ← permutation(N,C)
check([ ])       ←
check([Q|Qs])    ← ¬ attack(Q,Qs) ∧
                   check(Qs)
```

		Q	
Q			
			Q
	Q		

✓

$\downarrow_*$ **Derive automatically, by transformation**

```
queens(X,C)            ← queens(N,[ ],C)
queens(0,Qs,Qs)        ←
queens(N,SafeQs,Qs)    ← place(SafeQs,Q) ∧
                         ¬ attack(Q,SafeQs) ∧
                         M>0 ∧ M is N-1 ∧
                         queens(M,[Q|SafeQs],Qs)
```

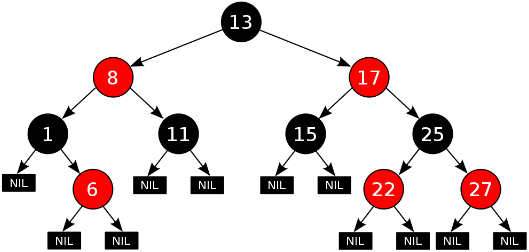
		Q	
Q			
			Q
	Q		

Enumeration of Complex Data-Structures

Red-Black Trees

Nodes are:

- colored (red or black)
- marked (by a key)



A binary tree satisfying the following invariants:

(I_1) **red** nodes have **black children**

(I_2) every **root-to-leaf path** has the same **number of black nodes**

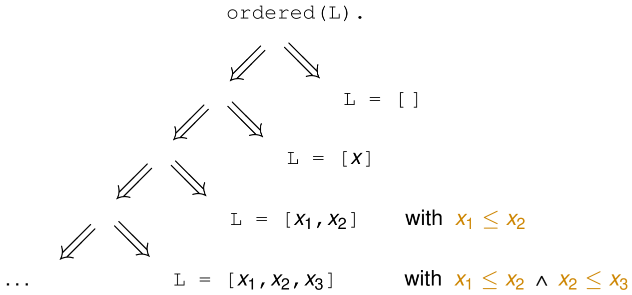
(I_3) keys are **ordered** left-to-right

imperative vs **declarative** languages for test-case generation

CLP Evaluation for Test Generation

ordered([])
ordered([x])
ordered([x_1, x_2 | L]) $\leftarrow \underbrace{x_1 \leq x_2}_{\text{solver for } \mathcal{T}} \wedge \underbrace{\texttt{ordered}([x_2 \mid L])}_{\text{resolution}}$

As a generator:



- Constraint-based
  [DeMillo-Offutt '91, Meudec (ATGen) '01, Euclide '09]
- Constraint Logic Programming -based
  [PathCrawler '05, Charreteur-Botella-Gotlieb '01, jPET '11]

A CLP-based Encoding of Red-Black Trees

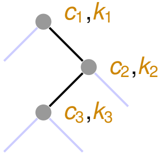
```
rbtree(T,MinSize,MaxSize,NumKeys) ←
    % Preamble
      ...DOMAINS...,
    % Symbolic Definition
      lbt(T,S,Keys,[]),                  % data structure shape
      pi(T,D), ci(T,Colors,[]),          % filters
      ordered(T,0,NumKeys),              % filters
    % Instantiation
      fd_labeling(Keys), fd_labeling(Colors).
```

lbt(T,S,Keys,[])	if	**T** is labeled binary tree with **S** nodes
pi(T,D)	if	the tree **T** satisfies the **path invariant**
ci(T,Colors)	if	the tree **T** satisfies the **color invariant**
ordered(T,0,NumKeys)	if	the labels (keys) in **T** are **ordered** left-to-right
fd_labeling(X)	if	the variable **X** is instantiated to a feasible value

Shape Rejection

`Tree ::= e | Color × Key × Tree × Tree` with `Color` in $\{0, 1\}$ (red, black) and `Key` in $\{0, \ldots, \text{MaxKey}\}$

Size 3 (a possible solution):



structure shape

$\wedge$

$$c_1 = c_1 + c_2 \;\wedge\; c_1 + c_2 = c_1 + c_2 + c_3$$

path invariant

$\wedge$

$$c_1 + c_2 > 0 \;\wedge\; c_2 + c_3 > 0$$

color invariant

$\wedge$

$$k_1 < k_2 \;\wedge\; k_1 < k_3 \;\wedge\; k_3 < k_2$$

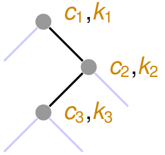
ordering

Shape Rejection

`Tree ::= e | Color` $\times$ `Key` $\times$ `Tree` $\times$ `Tree`

with `Color` in $\{0, 1\}$ (red, black)
and `Key` in $\{0, \ldots, \text{MaxKey}\}$

Size 3 (a possible solution):



$\bigwedge$

$$c_1 = c_1 + c_2 \;\wedge\; c_1 + c_2 = c_1 + c_2 + c_3$$

$\bigwedge$

$$c_1 + c_2 > 0 \;\wedge\; c_2 + c_3 > 0$$

$\bigwedge$

$$k_1 < k_2 \;\wedge\; k_1 < k_3 \;\wedge\; k_3 < k_2$$

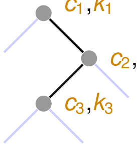
structure shape | path invariant | color invariant | ordering

lbt(T,S,Keys,[ ]) $\wedge$ pi(T,D) $\wedge$ ci(T,Colors) $\wedge$ ordered(T,...)

Shape Rejection

`Tree ::= e | Color × Key × Tree × Tree` with **Color** in $\{0, 1\}$ (red, black) and **Key** in $\{0, \ldots, \text{MaxKey}\}$

Size 3 (a possible solution):



$\bigwedge$

$$c_1 = c_1 + c_2 \;\wedge\; c_1 + c_2 = c_1 + c_2 + c_3$$

$\bigwedge$

$$c_1 + c_2 > 0 \;\wedge\; c_2 + c_3 > 0$$

$\bigwedge$

$$k_1 < k_2 \;\wedge\; k_1 < k_3 \;\wedge\; k_3 < k_2$$

UNFEASIBLE

structure shape — path invariant — color invariant — ordering

lbt(T,S,Keys,[ ]) $\wedge$ pi(T,D) $\wedge$ ci(T,Colors) $\wedge$ ordered(T,...)

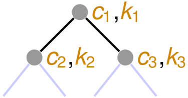
No instantiation possible $\Rightarrow c_2 = 0 \wedge c_3 = 0$

Shape Concretization

`Tree ::= e |` **Color** × **Key** × `Tree` × `Tree`

with **Color** in $\{0, 1\}$ (red, black)
and **Key** in $\{0, \ldots, \text{MaxKey}\}$

Size 3 (another solution) :



$$\bigwedge \quad c_1 + c_2 = c_1 + c_3 \quad \bigwedge \quad \begin{array}{c} c_1 + c_2 > 0 \\ \wedge \\ c_1 + c_3 > 0 \end{array} \quad \bigwedge \quad \begin{array}{c} k_2 < k_1 \\ \wedge \\ k_1 < k_3 \end{array}$$

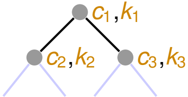
FEASIBLE

Instantiations:

Shape Concretization

`Tree ::= e | Color × Key × Tree × Tree` with **`Color`** in $\{0, 1\}$ (red, black) and **`Key`** in $\{0, \ldots, \text{MaxKey}\}$

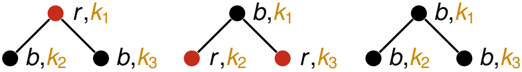
Size 3 (another solution) :



$$\bigwedge \quad c_1 + c_2 = c_1 + c_3 \quad \bigwedge \quad \begin{matrix} c_1 + c_2 > 0 \\ \wedge \\ c_1 + c_3 > 0 \end{matrix} \quad \bigwedge \quad \begin{matrix} k_2 < k_1 \\ \wedge \\ k_1 < k_3 \end{matrix}$$

FEASIBLE

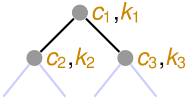
Instantiations:



Shape Concretization

`Tree ::= e | Color × Key × Tree × Tree` with **Color** in $\{0, 1\}$ (red, black) and **Key** in $\{0, \dots, \text{MaxKey}\}$

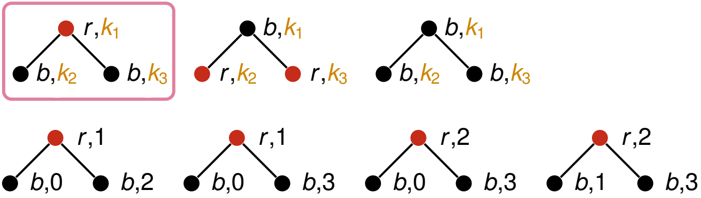
Size 3 (another solution) :



$$\wedge \quad c_1 + c_2 = c_1 + c_3 \quad \wedge \quad \begin{matrix} c_1 + c_2 > 0 \\ \wedge \\ c_1 + c_3 > 0 \end{matrix} \quad \wedge \quad \begin{matrix} k_2 < k_1 \\ \wedge \\ k_1 < k_3 \end{matrix}$$

FEASIBLE

Instantiations:

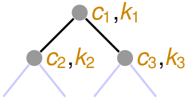


Shape Concretization

`Tree ::= e |` **Color** × **Key** × `Tree` × `Tree`

with **Color** in $\{0, 1\}$ (red, black)
and **Key** in $\{0, \ldots, \text{MaxKey}\}$

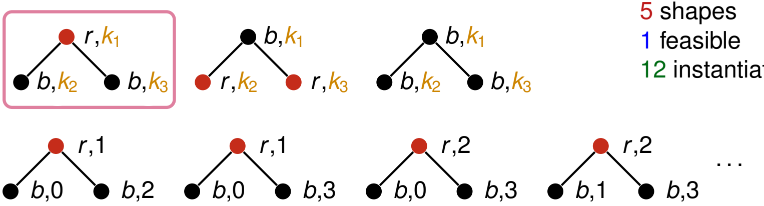
Size 3 (another solution) :



$\bigwedge \quad c_1 + c_2 = c_1 + c_3 \quad \bigwedge \quad \begin{matrix} c_1 + c_2 > 0 \\ \wedge \\ c_1 + c_3 > 0 \end{matrix} \quad \bigwedge \quad \begin{matrix} k_2 < k_1 \\ \wedge \\ k_1 < k_3 \end{matrix}$

FEASIBLE

Instantiations:

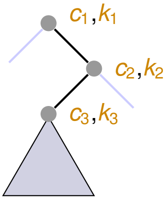


5 shapes
1 feasible
12 instantiations

we let the solver choose an instantiation order to minimize backtracking

Optimizing Generators by Transformation

Size 20



No way of placing the remaining 17 nodes to build a feasible tree

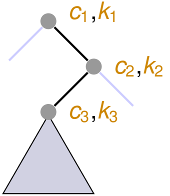
$c_1 = c_1 + c_2 \wedge c_1 = c_1 + c_2 + c_3 + X \wedge$
$X \geq 0 \wedge c_1 + c_2 > 0 \wedge c_2 + c_3 > 0$ UNFEASIBLE

Yet, there would be 35357670 **shapes** (and corresponding feasiblity tests)

Idea: apply filter earlier

Optimizing Generators by Transformation

Size 20



No way of placing the remaining
17 nodes to build a feasible tree

$$c_1 = c_1 + c_2 \wedge c_1 = c_1 + c_2 + c_3 + X \wedge$$
$$X \geq 0 \wedge c_1 + c_2 > 0 \wedge c_2 + c_3 > 0$$

UNFEASIBLE

Yet, there would be 35357670 **shapes**
(and corresponding feasiblity tests)

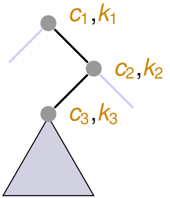
Idea: apply filter earlier

The Synchronization transformation strategy

- automates optimization
- **filter promotion + tupling + folding** (local optimization inductively propagated)
- force the generator to have the desired behavior
- reduces don't care nondeterminism (less **failures**)
- guided by the **inductive traversal** of (a slice of) the data-structure

Optimizing Generators by Transformation

Size 20



No way of placing the remaining
17 nodes to build a feasible tree

$c_1 = c_1 + c_2 \wedge c_1 = c_1 + c_2 + c_3 + X \wedge$
$X \geq 0 \wedge c_1 + c_2 > 0 \wedge c_2 + c_3 > 0$ UNFEASIBLE

Yet, there would be 35357670 **shapes**
(and corresponding feasiblity tests)

Idea: apply filter earlier

The Synchronization transformation strategy

- automates optimization
- **filter promotion + tupling + folding** (local optimization inductively propagated)
- force the generator to have the desired behavior
- reduces don't care nondeterminism (less **failures**)
- guided by the **inductive traversal** of (a slice of) the data-structure

Related techniques : compiling control, co-routining

Experiments

Size	***RB Trees***	Time		
		Original	Synchronized	Korat
6	20	0	0	0.47
7	35	0.01	0	0.63
8	64	0.02	0	1.49
9	122	0.08	0	4.51
10	260	0.29	0.01	21.14
11	586	1.07	0.03	116.17
12	1296	3.98	0.06	-
13	2708	14.85	0.12	-
14	5400	55.77	0.26	-
15	10468	-	0.55	-
...	...	...	...	...
20	279264	-	25.90	-

Size = number of nodes
RB Trees = number of structures found
Time = (in seconds) for generating all the structures

Zero means less than 10 ms, and
(-) means more than 200 seconds

Summarizing

- improved **declarativeness**
- heaparrays, disjoint sets, various lists/trees, ...
- baseline good, room for optimization
- a language of **composable**/**refinable** generators
- automatic extraction of CLP generators from **contracts** (a form of program synthesis)
- graph-like structures