

Constraints for (Parameterized) Verification

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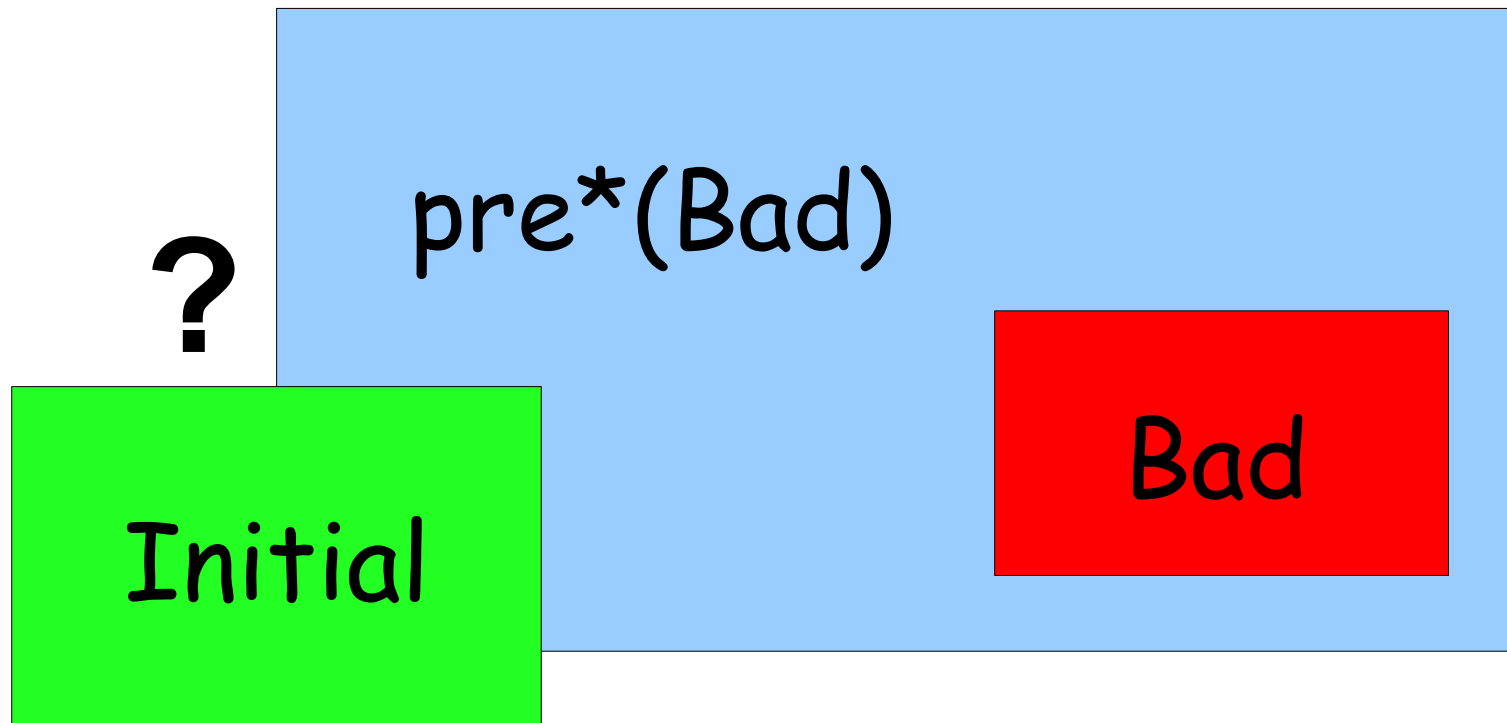


Model Checking

- Ingredients of **Model Checking**
 - Operations on **sets of states**:
union, intersection, etc
 - pre/post** operator:
transformation of sets of states
 - **Fixpoint computation**:
pre* and post*
:
(transitive closure of transition relation)

Safety: General Framework

- Verification of safety properties can be reduced to reachability of bad states



Symbolic Model Checking

- Ingredients of Symbolic Model Checking
 - **Symbolic** operations on sets of states:
union, intersection, etc
 - pre/post operations:
Symbolic transformation of sets of states
 - Fixpoint computation:
pre* and post* **symbolic** computations

Symbolic Model Checking

- For finite state systems:
 - BDDs/Boolean Formulas
- For **reachability** in infinite-state systems
 - Regions of timed automata/Zones
[Alur-Dill, Abdulla et al.]
 - Finite-state automata for pushdown systems
[Bouajjani-Esparza-Maler,...]:
 - ...

Constraints 4 Safety

- Metaphor of constraints to generalize the role of BDDs in symbolic model checking
- General requirements to obtain effective and terminating procedures to check reachability
- Focus on constraints for systems composed of an arbitrary but finite number of component (**Parameterized Verification**)

[Fribourg, Delzanno-Podelski, Abdulla-Jonsson,...]

Constraints 4 Safety

Consider a system with set of states Q ,

A constraint system $(C, <)$ is such that

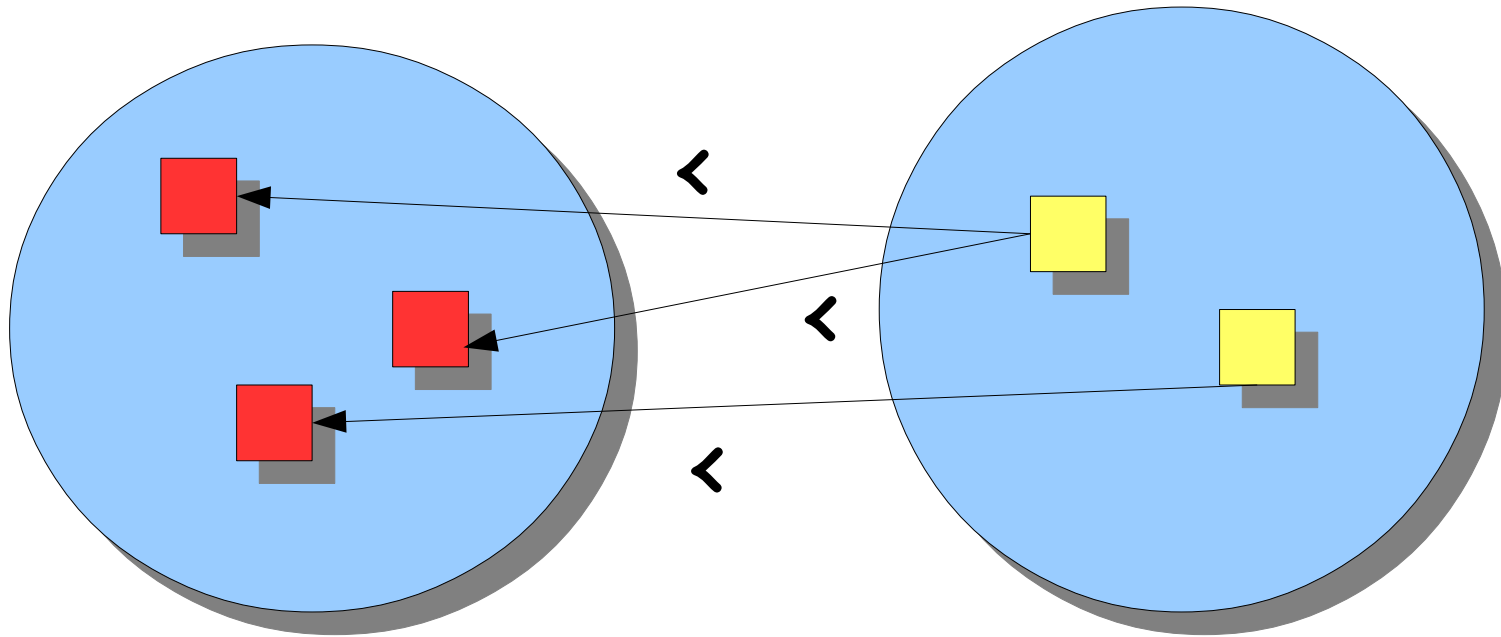
(denotation) $[c]$ is a subset of Q for c in C

(entailment) $c < d$ implies $[d]$ is contained into $[c]$

[Abdulla-Cerans-Jonsson-Tsay, Abdulla-Jonsson]

Finite sets of constraints

$$S \ll S'$$



$S \ll S'$ iff for each d in S' there exists c in S
s.t. $c < d$

Ingredients for Reachability

- Representation of **initial/bad** states (I and B)
- **Decidable** entailment test \prec
- **Algorithm for computing predecessors**, i.e.,
$$\text{Pre}(S) = S' \text{ s.t. } [S'] = \text{pre}([S])$$

⋮
- Decidable test "I intersects S"

Naive Backward Reachability

1. $R := B$; (set of constraints \times bad states)
2. $O := R$; (to check stability)
3. $R := (R \text{ union } \text{Pre}(R))$;
4. If $(O \ll R)$ return (I intersects R)
5. goto 1.

:

Some Optimizations

- If d in $\text{Pre}(c)$ first check if $c < d$ then compare with the remaining constraints in O
- Eliminate redundant constraints in O , i.e., all constraints S that are subsumed by new constraints (and constraints that generated S)
- Specific strategies for computing Pre (e.g. always try to compute first the "more" general constraints perhaps using a mix dfs and bfs)
- ...

Ensuring Termination

- $(C, <)$ is a well quasi ordering (**wqo**) iff for every **sequence** of constraints $c_1 c_2 c_3 \dots$ there exist $i < j$ s.t. $c_i < c_j$
- If $(C, <)$ is a wqo, then every **chain** $S_1 S_2 \dots$ (e.g. sets computed during backward reachability) eventually **stabilizes**, i.e., there exists k s.t. $[S_k] = [S_{k+1}]$

Note: it only works for chains

Petri Nets

- k counters, $++$, $--$, no zero-test
 - Configurations are vectors of natural numbers
 - $m < m'$ if $m(p)$ is less or equal than $m'(p)$ for every place p (wqo by Dickson's lemma)
- We can solve coverability:
 - Given m and m' , starting from m can we reach m'' s.t. $m' < m''$?

Termination is guaranteed!

The Role of Constraint Solvers

- Constraint solvers can be used as engine for:
 - Computing Pre (renaming/projection)
 - Check entailment
 - Check intersection with initial states
- Observation:
 - In general we need sets of constraints (disjunctions) we may work with approximations
 - Termination guarantees vs practical termination

Some examples

- ALV: BDD+Omega [UCSB]
- CLP-based model checking: clp(R) [MPI]
- HyTech, PHaver: Polyhedra Lib, Parma Polyhedra Lib (PPL)
- Sharing Trees, Interval Sharing Trees [ULB]
- Combined representations TRES [Liafa]
- Automata for queues and integers [Liege]
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Parameterized Verification

(coined by Sistla?)

Parameterized Verification

- Goal: verify safety for systems composed of an arbitrary but finite number of components
 - Petri nets (abstractions of multithreaded programs)
 - Broadcast protocols (abstractions of cache coherence protocols)
 - Skeletons of concurrent/distributed algorithms
 - Network protocols with different topologies (flooding, routing)

Several Approaches

- Invariants and theorem proving
- Abstractions and finite models (cut off points)
- Regular model checking
(sets of configurations=automata)
- Forward/backward reachability

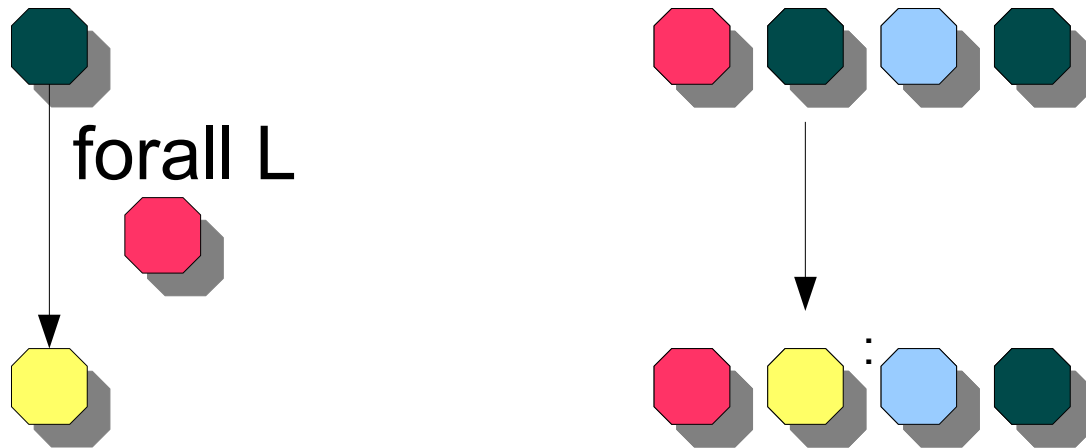
Focus: constraint-based approach

Linearly Ordered Systems

- A system is defined as an (unbounded) array of processes
- Each process is defined by an automata with guards
- Guards have the form:
 - Exists a process to the left/right with state q
 - All processes to the left/right have states in S
- We also consider rendez-vous and broadcast
- Examples: mutex and coherence protocols with atomic guards

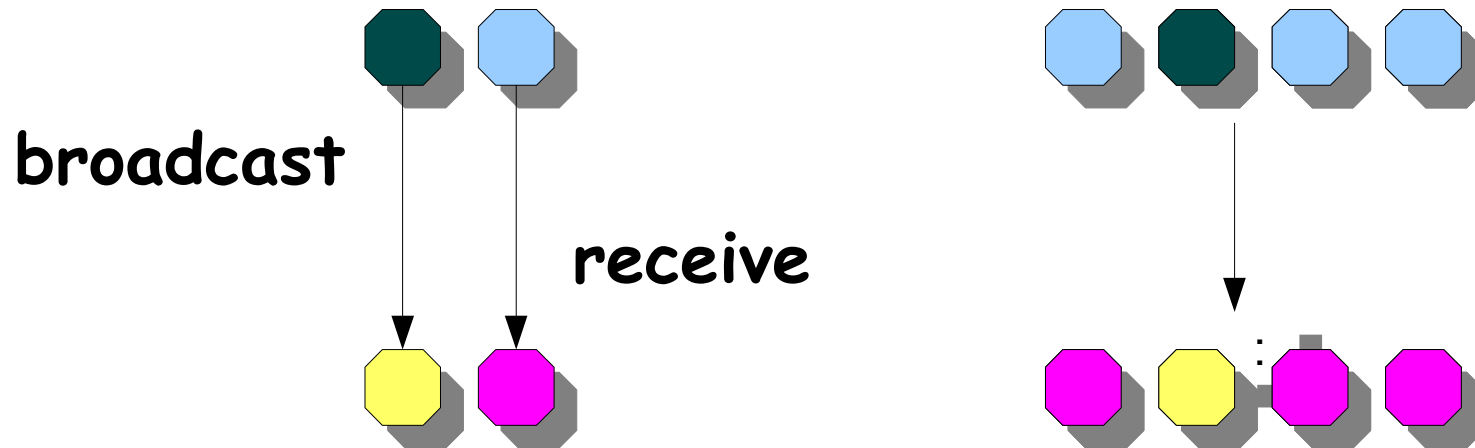
Semantics

- Configurations: words over a finite alphabet
- Transitions: word2word transformation
- E.g. with 4 processes



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Safety

- **Mutual exclusion**: Reachable configuration contains at most one occurrence of state q (=critical section)
- **Bad states**: all configurations with two or more occurrences of state q
- Bad states are upward closed w.r.t. subword ordering (they are generated by the word qq)
As a regular expression: $Q^*qQ^*qQ^*$

Analysis

- The use of universal quantification makes the model Turing complete
- Precise analysis --> no termination:
automata/regular expressions as symbolic representations of sets of states
- Approximate analysis --> termination:
upward closed sets of words

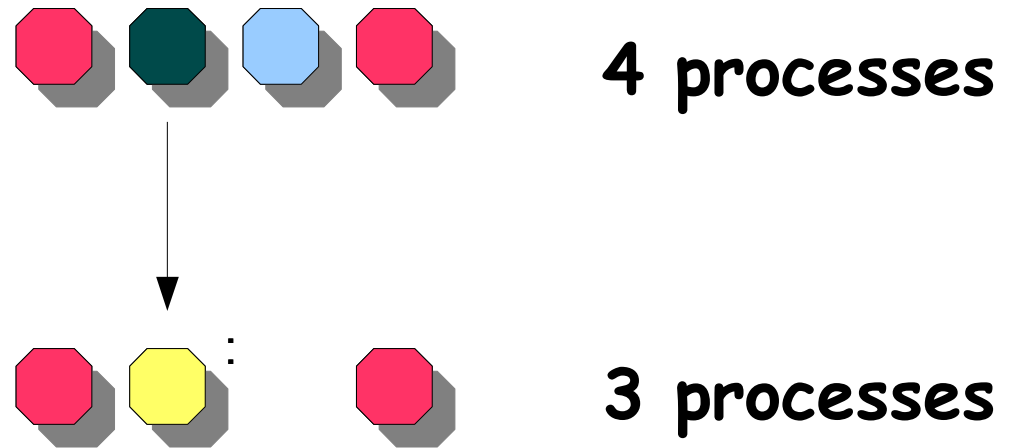
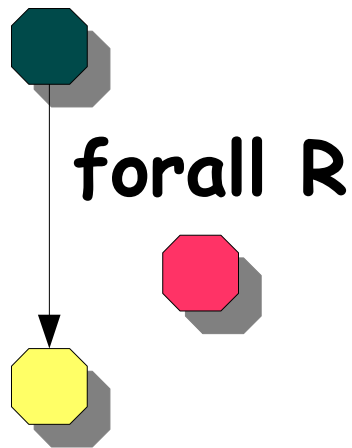
A Simple Abstraction

- We work with upward closed sets of words
- Finitely generated because subword is a wqo
- Constraint: a set of words B
 $[B] = \{w' \mid w \text{ subword of } w', w \text{ in } B\}$
- **Approximation:**
Starting from an upward closed set S we compute the minimal upward closed set that contains $\text{pre}(S)$

[Monotonic Abstraction: Abdulla, Rezine,....]

A Simple Abstraction

- Operationally the abstraction corresponds to cancellation of processes that do not satisfy the guard:



Properties

- Backward reachability is guaranteed to terminate (subword is a wqo)
- Simple but it verifies safety on most of the examples of linearly ordered systems with atomic guards in the literature of parameterized systems

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Spurious Traces

- It fails on
 - Ordered systems where processes in certain states act as sentinels (e.g. Szymanski's alg.)
 - Unordered systems (counter systems) in which there are variables that keep track of processes (e.g. readers/writers)
- In this examples upward closed sets are too rough :

Patch for Ordered Systems

- We need to keep information coming from **universal quantification**
- We use r.e. $(a_1 \dots a_n, P) = P^* a_1 P^* \dots P^* a_n P^*$
 P is contained in Q , a_1, \dots, a_n in P
- New wqo: $(w, P) < (w', P')$ iff
 w subword of w' , P' is contained into P
- Idea: when computing Pre for a rule with guard forall S , $(w, P) \rightarrow (w', P \text{ intersect } S)$

[Delzanno-Rezine]

Cegar for Unordered Systems

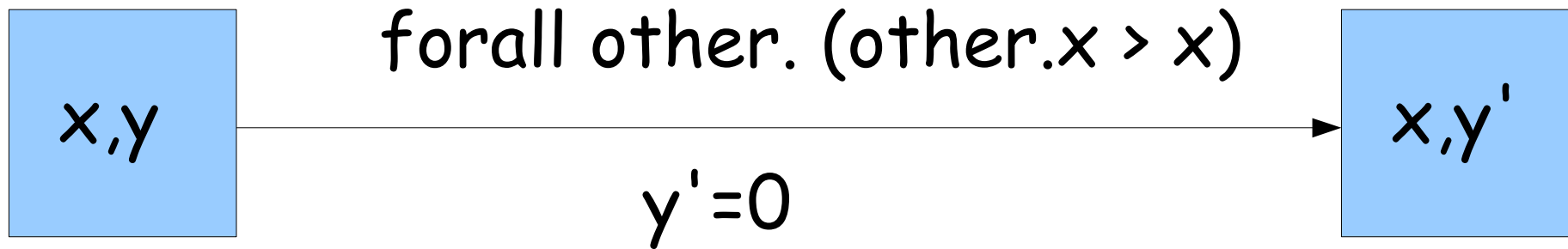
- For unordered systems we have defined an automatic refinement of the ordering
- The refinement computes **interpolants** for a pair of constraints in an abstract trace for which there exists no concrete transition connecting them
- Ordered case still open ...

Infinite-state processes

- Each process has a finite number of local variables ranging over integers
- Existential and universal global conditions where we compare variables of different processes
- Examples, e.g., Lamport's mutex (every process has local integer variables)
- Goal: try to verify mutual exclusion for any number of processes

Infinite-state processes

- Models = automata with data variables and global guards



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Constraints?

We can use formulas over processes and relation over data

$$c = p(\text{think}, X), p(\text{wait}, Y), X < Y$$

Denotation: Upward closure w.r.t. multiset inclusion of all possible instances of $p(\text{think}, X), p(\text{wait}, Y)$ obtained by taking solutions of $X < Y$

⋮

$p(\text{think}, 1), p(\text{wait}, 2), p(\text{think}, 2)$ belongs to $[c]$

Constraint solving?

- Satisfiability:
we check satisfiability of numerical constraint
- Entailment:
 - injection of processes,
 - entailment of terms, :
 - unification, projection and constraint entailment

Entailment

$$c = p(S, X), p(\text{wait}, Y), X < Y$$



$$d = p(\text{think}, X'), p(\text{wait}, Y'), p(\text{use}, T'), X' < T', T' < Y'$$

- S subsumes think

- $X < Y$ subsumes

Exists T' . $X' < T', T' < Y', X = X', Y = Y'$

- $[d]$ is contained into $[c]$

Non atomic Guards?

Non atomic guards are modelled via marking subprotocols (keep track of checked processes)

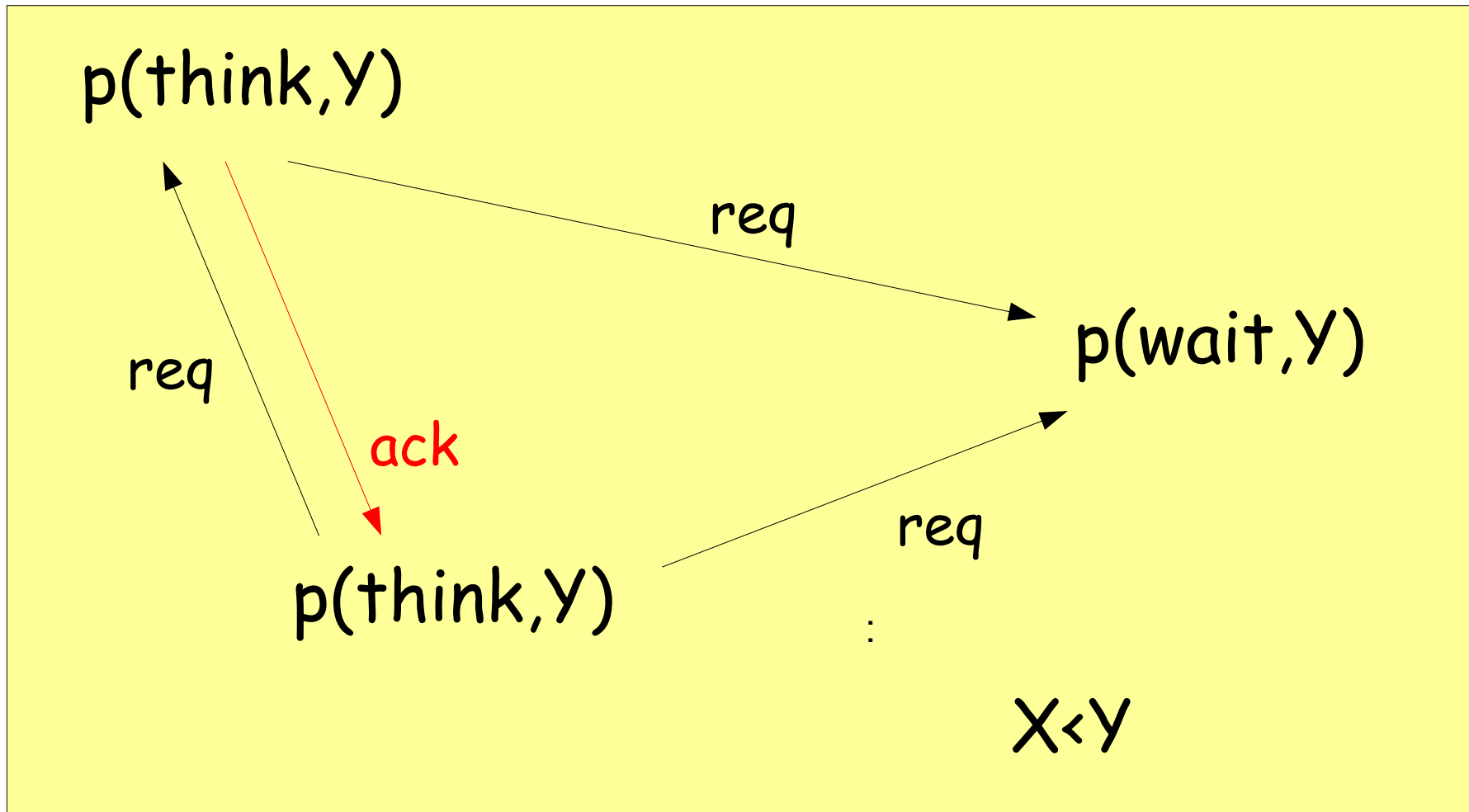
forall other. other.X > self.X

becomes

P_i : Send req(X) to every other process

P_j : Receive req(V) from P_i ;
if $X > V$ send(ack, P_i)

Constraints for non atomic guards = graphs



Abstractions and Termination

- We can still apply monotonic abstraction to work with upward closed sets (i.e. represent Pre as a finite sets of our constraints)
- Termination guarantees for special cases:
 - Guards are **gap-order** constraints
 - Each processes has at most 1 local variable

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Gap order: $x+c < y$ where c is a natural number

Implementation/results

- We have implemented ad hoc solvers in CLP(R) (to exploit unification and constraint solving) and PPL (Parma Polyhedra Lib) (to combine different symbolic representations like BDDs and constraints)
- We could verify safety for classical algorithms like Lamport's dist mutex, and Ricart Agrawala
- Non atomic Szymanski is still open

Other approaches

- Invariant checking with rich theories
- Forward + accelerations for counter systems and well-structured transition systems
- Static and dynamic cut-off points (try 2,3,4 processes + generalization)
- SMT solvers as constraint engine
- Program Transformations
- Theorem proving (Finite model generators)

Current work

- Non atomic case → use of graphs
- Graph-based tools for network protocols (routing, broadcast)
- We are studying the properties of models: for which operations/classes of graphs we can solve problems like coverability

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Some other applications

- Parameterized verification for biological systems
 - Bioambients and P-systems (tree structures and petri nets)
 - Confromon P-systems (petri nets + energy)
 - Kappa calculus (graph-based rules)

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