Generalised Spatial and Temporal Placement Constraint: Current Status and Evolution

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M. Sbihi, C. Truchet, S. Zampelli (**LINA**)

CONTEXT

We present a **spatio-temporal** global constraint kernel *geost*.
Initially developped in the ongoing EU project NetWMS (by **LINA** and **SICS**).
Inital focus was the geometrical aspect (*even if time was introduced*).

Extended in some project starting in 2009 (SelfXL).

WHAT IS IT ALL ABOUT ?

Dynamic problems which mix geometrical and temporal aspects.

Introduction

External Geometrical Constraints

Internal Geometrical Constraints

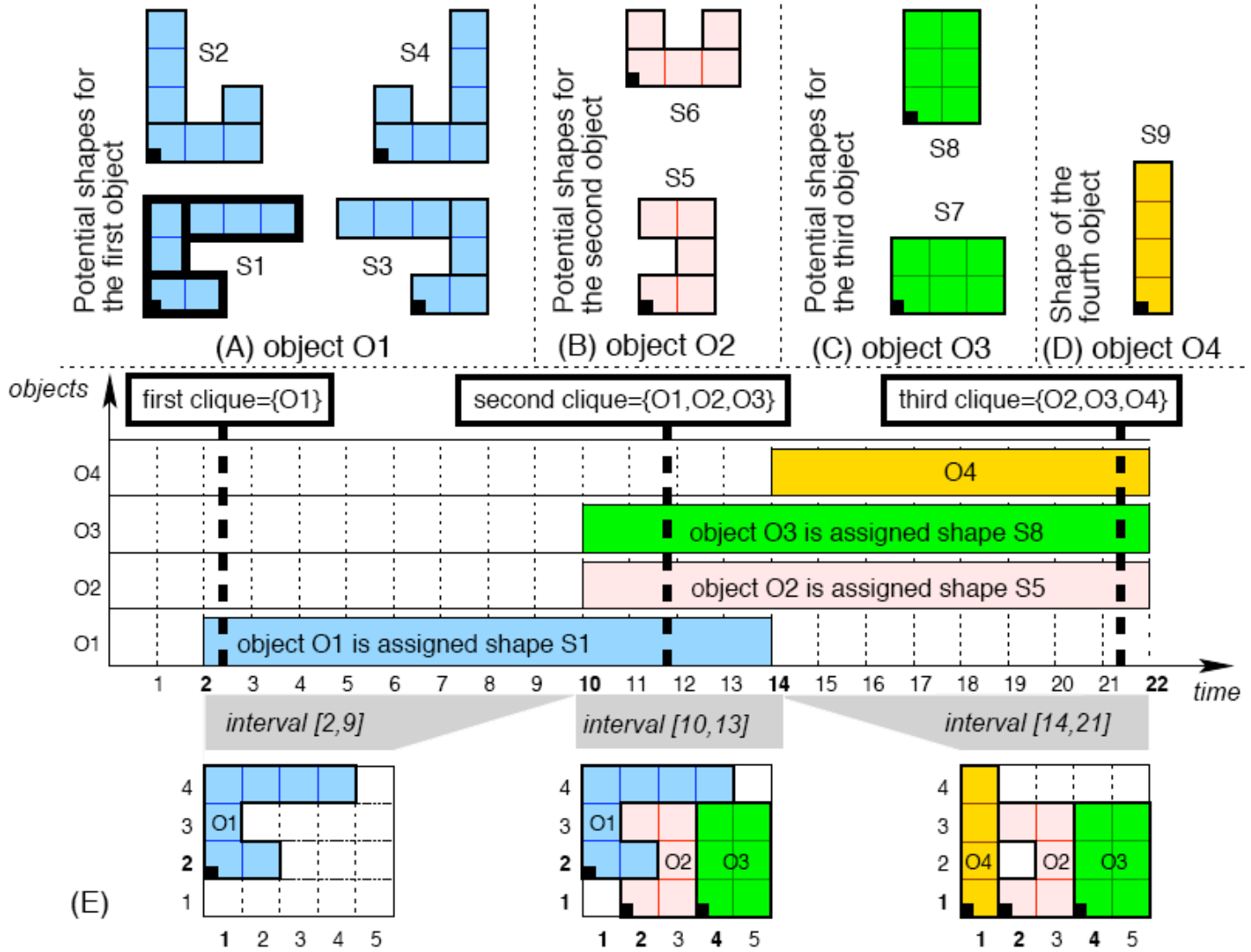
The Propagation Kernel

A First Evaluation

Time (and trajectories)

geost on the web

A Generic Placement Kernel: *geost*



A Generic Placement Kernel: *geost*

```
geost(2,
```

Number of dimensions

```
[object(1,1,[1,2], 2,12,14),
 object(2,5,[2,1],10,12,22),
 object(3,8,[4,1],10,12,22),
 object(4,9,[1,1],14, 8,22)],
```

Object Id, Shape Id, Origin, Start, Duration, End

Additional attributes (type, weigth, customer, …) can eventually be added

Objects

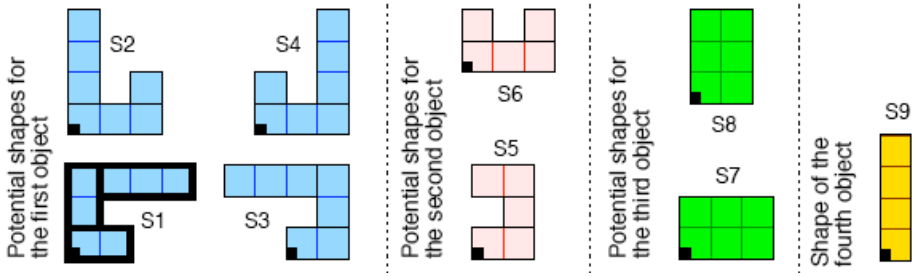
```
 sbox(1,[0,0],[2,1]), sbox(1,[0,1],[1,2]), sbox(1,[1,2],[3,1]),
 sbox(2,[0,0],[3,1]), sbox(2,[0,1],[1,3]), sbox(2,[2,1],[1,1]),
 sbox(3,[0,0],[2,1]), sbox(3,[1,1],[1,2]), sbox(3,[2,2],[3,1]),
 sbox(4,[0,0],[3,1]), sbox(4,[0,1],[1,1]), sbox(4,[2,1],[1,3]),
 sbox(5,[0,0],[2,1]), sbox(5,[1,1],[1,1]), sbox(5,[0,2],[2,1]),
 sbox(6,[0,0],[3,1]), sbox(6,[0,1],[1,1]), sbox(6,[2,1],[1,1]),
 sbox(7,[0,0],[3,2]),
 sbox(8,[0,0],[2,3]),
 sbox(9,[0,0],[1,4])],
```

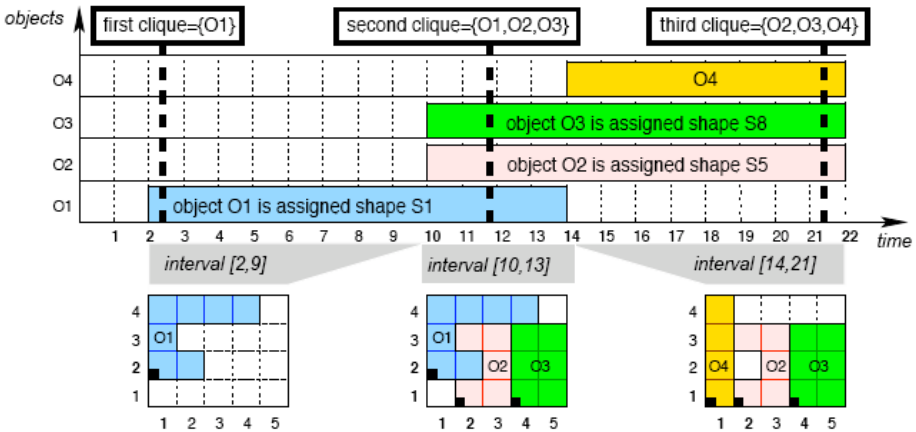
Potential shapes,
where a shape is defined by a set of sboxes sharing the same shape id

```
[non-overlapping([0,1],[1,2,3,4]),included([0,1],[1,2,3,4],[1,1],[5,4])])
```

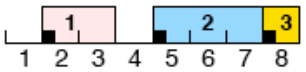
Attributes Objects Attributes Objects Box

List of external constraints

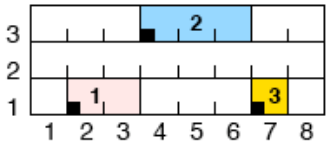




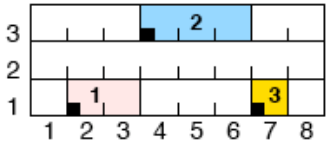
Applications

(A) 

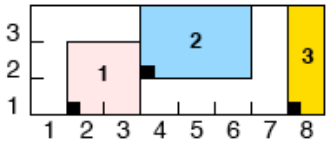
disjunctive

(B) 

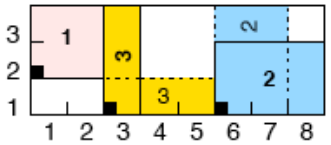
machine assignment
(e.g., *parking assignment*)

(C) 

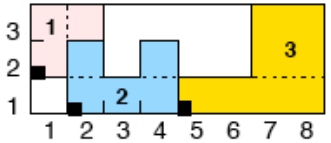
machine assignment
(machine dependant duration)

(D) 

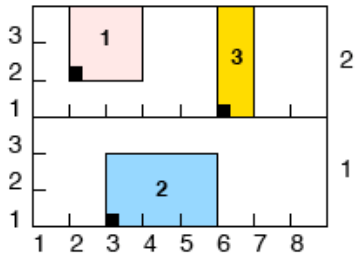
2D non-overlapping
(fixed orientation)

(E) 

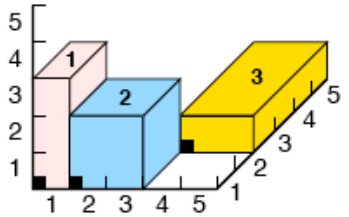
2D non-overlapping
(90° rotation)

(F) 

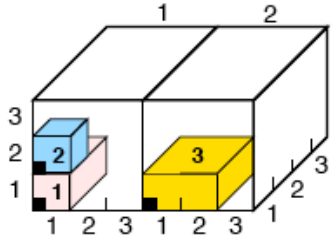
2D non-overlapping
(irregular shapes)

(G) 

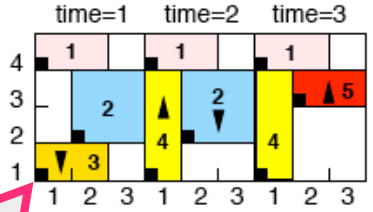
2D non-overlapping
and assignment

(H) 

3D non-overlapping

(I) 

3D non-overlapping
and assignment

(J) 

pick-up delivery
(time dimension)

```
geost(2, [object(1,1,[1,4]),1,3,4), object(2,2,[2,2]),1,2,3), object(3,1,[1,1]),1,1,2),
          object(4,3,[1,1]),2,2,4), object(5,1,[2,3]),3,1,4)],
         [sbox(1,[0,0],[2,1]), sbox(2,[0,0],[2,2]), sbox(3,[0,0],[1,3])],
         [non-overlapping([0,1],[1,2,3,4,5])] )
```

Mixing Constraints on Several Dimensions

EXAMPLE OF PROBLEM

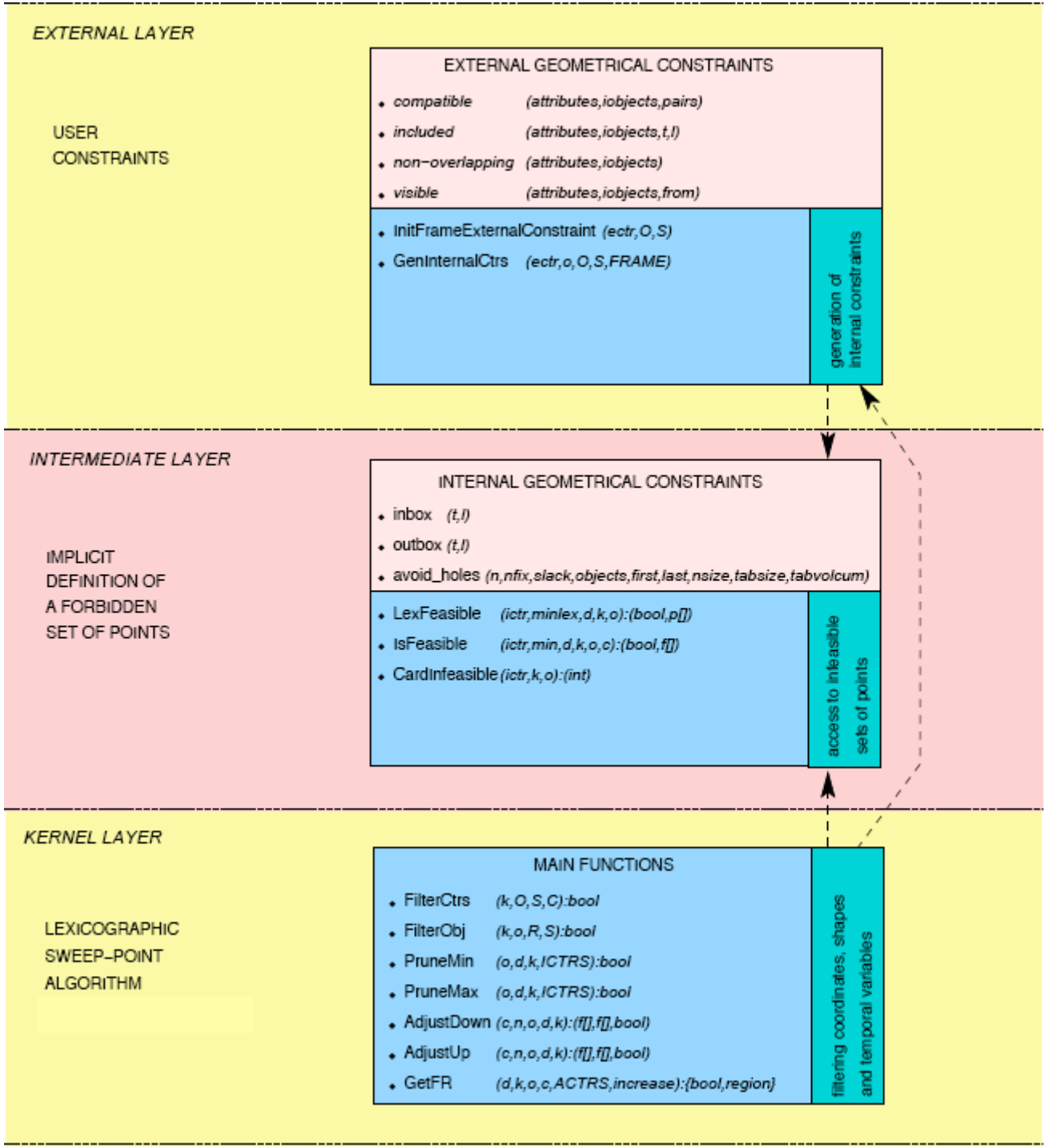
Input: A set of parallelepipeds *P* and a subset *P'* of *P*

Constraints: (1) all parallelepipeds of *P* should not overlap
(2) no parallelepipeds of *P'* should be piled

Solution with *geost*: $non\text{-}overlapping([0,1,2],\mathcal{P})$

$non\text{-}overlapping([0,1],\mathcal{P}')$

Overall Architecture



constraints

functions

purpose

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geost on the web

Example of External Constraint: *distance*

Depending of the norm we consider we have different distances:

ℓ_q norm, $q \geq 1$ $\quad \|x\|_q = (\sum_{i=0}^{k-1} |x[i]|^q)^{\frac{1}{q}}$

q=1: Manhattan
q=2: Euclidean

ℓ_∞ norm $\quad \|x\|_\infty = \max_{0 \leq i < k} |x[i]|$

Handle minimum and maximum distances between objects of *geost*
(*manipulate only integers !*)

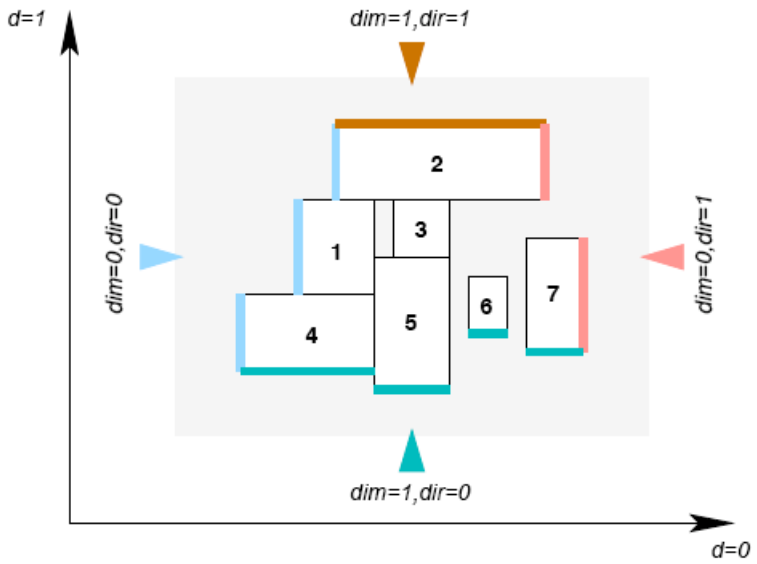
More general distances also consider the assignement dimension

Example of External Constraint: *visible*

IDEA

Given a set of potential observations places P,
and given for each box a set of visible faces,
the **visible** *constraint specifies that at least one visible face of each box*
should be entirely visible from at least one observation place of P
at the start and end(-1) time associated to the box.

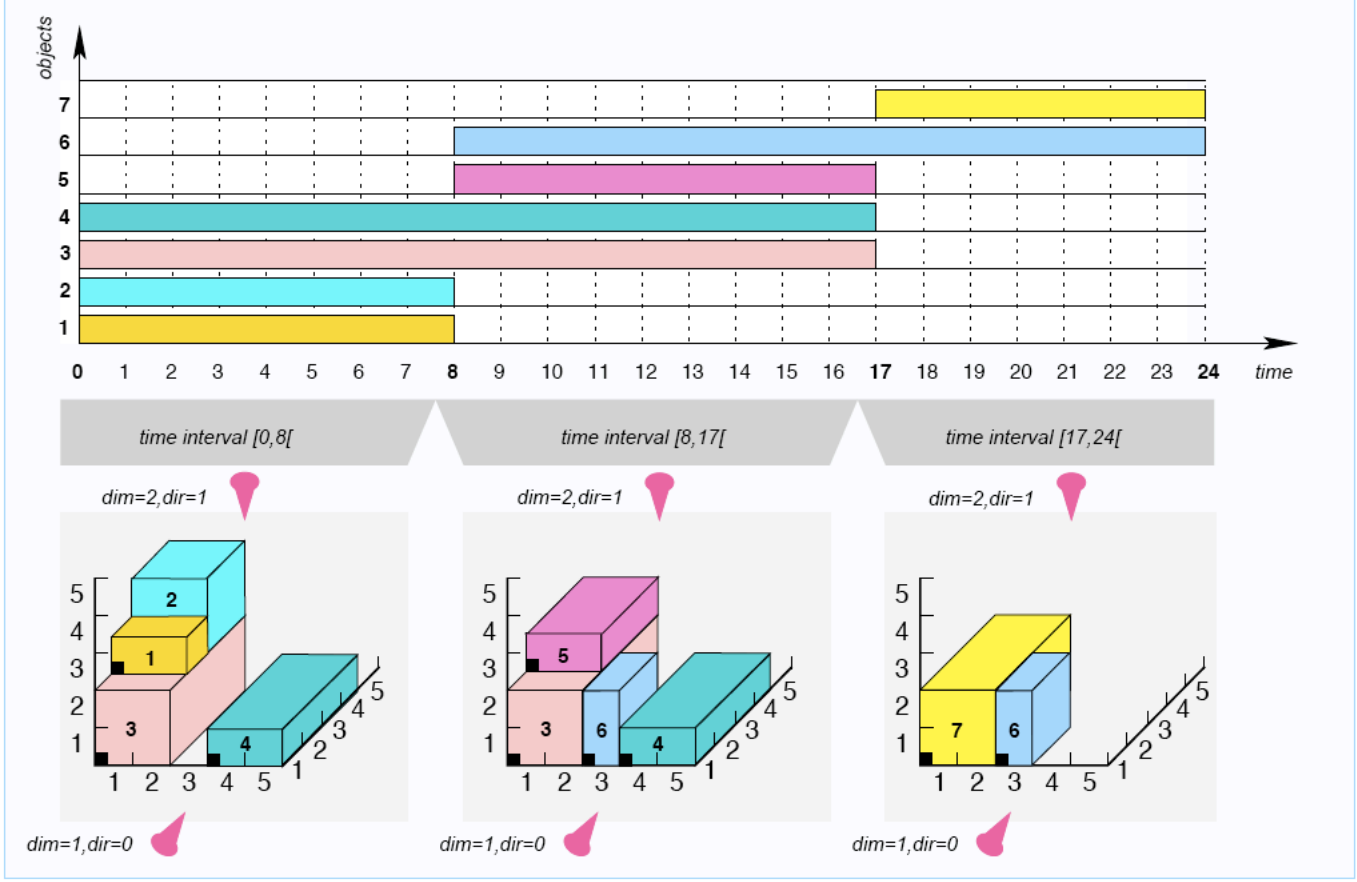
Completely visible faces
from a set of observations points



Application of *visible*: pick-up delivery

time-dimension again !

```
geost(3, [object(1,1,[1,2,3],  0,  8,  8), object(2,2,[1,3,3],0,8,  8), object(3,3,[1,1,1],0,17,17),
          object(4,4,[4,1,1],  0,17,17), object(5,5,[1,2,3],8,9,17), object(6,6,[3,1,1],8,12,24),
          object(7,3,[1,1,1],17,  7,24)],
         [shape(1,[0,0,0],[2,1,1],face–[<1,0>,<2,1>]), shape(2,[0,0,0],[2,2,2],face–[<1,0>,<2,1>]),
          shape(3,[0,0,0],[2,4,2],face–[<1,0>,<2,1>]), shape(4,[0,0,0],[2,4,1],face–[<1,0>,<2,1>]),
          shape(5,[0,0,0],[2,3,1],face–[<1,0>,<2,1>]), shape(6,[0,0,0],[1,2,2],face–[<1,0>,<2,1>])],
         [non–overlapping([0,1,2],[1,2,3,4,5,6,7]),
          visible([0,1,2,face],[1,2,3,4,5,6,7],[<1,0>]), visible([0,1,2,face],[1,2,3,4,5,6,7],[<2,1>])] )
```



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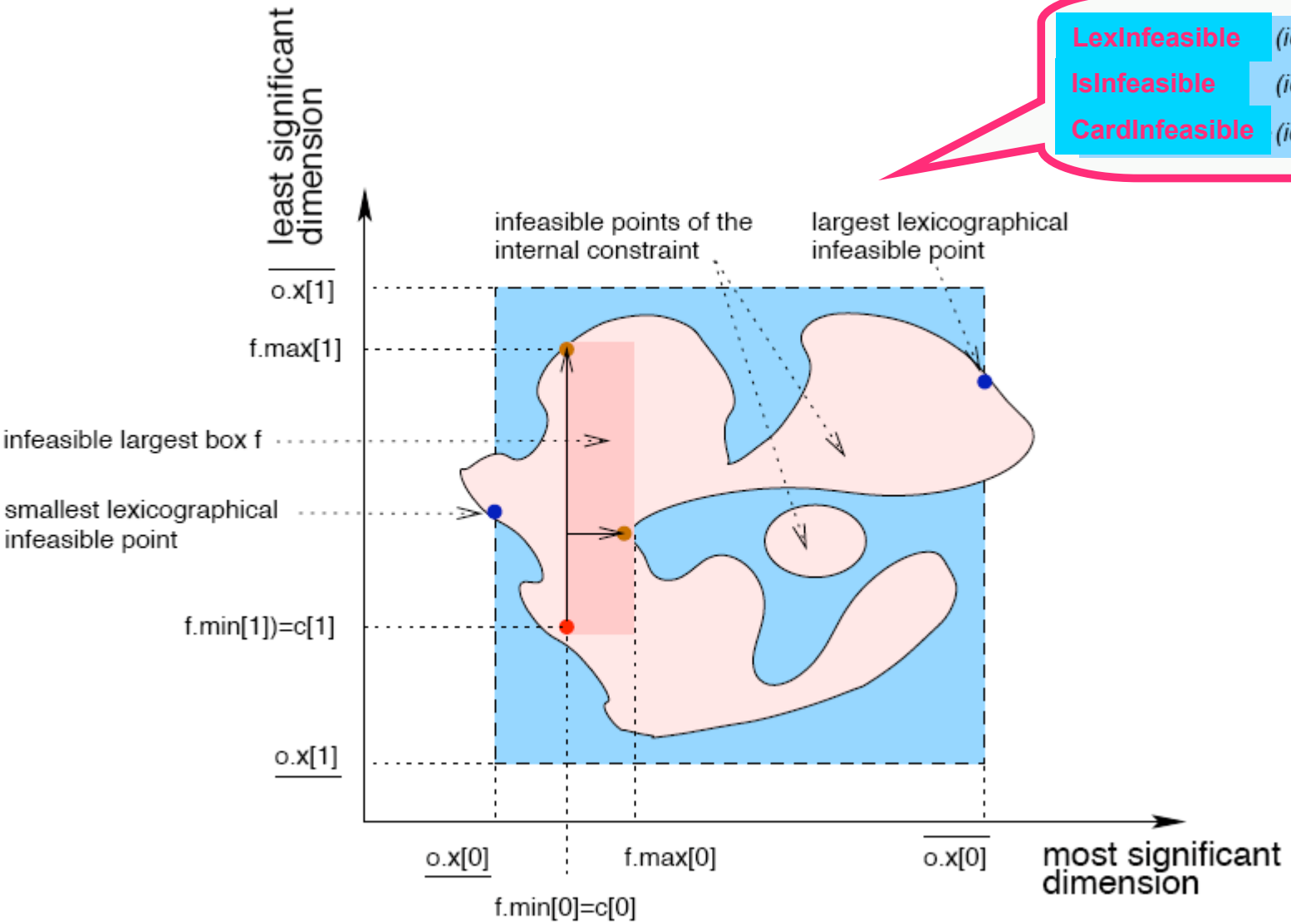
Intermediate Layer

SERVICES ASSOCIATED TO AN INTERNAL CONSTRAINT (i.e., a set of forbidden points)

LexInfeasible	*(ictr,minlex,d,k,o):(bool,p[])*
IsInfeasible	*(ictr,min,d,k,o,c):(bool,f[])*
CardInfeasible	*(ictr,k,o):(int)*



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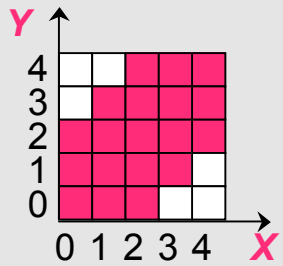
Communication between Constraints

SLOGAN OF CONSTRAINT PROGRAMMING

Constraints **communicate only** via the domains of their shared variables.

AN OTHER APPROACH

A constraint can be assimilated as a **set of forbidden points**, each variable corresponding to a dimension



$0 \leq X \leq 4 \quad 0 \leq Y \leq 4$

$|X-Y|>2$

PROBLEM: hard to **aggregate** sets of forbidden points associated to **different** constraints !!!

SOLUTION: set of forbidden points associated to different constraint should communicate
everything is handled in an implicit way (**lazy evaluation**)

Sweep Algorithms in Computational Geometry

Standard technique for comming up with **efficient** algorithms

- Computational geometry, an introduction

  [Preparata & Shamos, 1985]

- Computational Geometry, Algorithms and Applications

  [Berg, Kreveld, Overmars & Schwarzkopf, 1997]

- Géométrie algorithmique

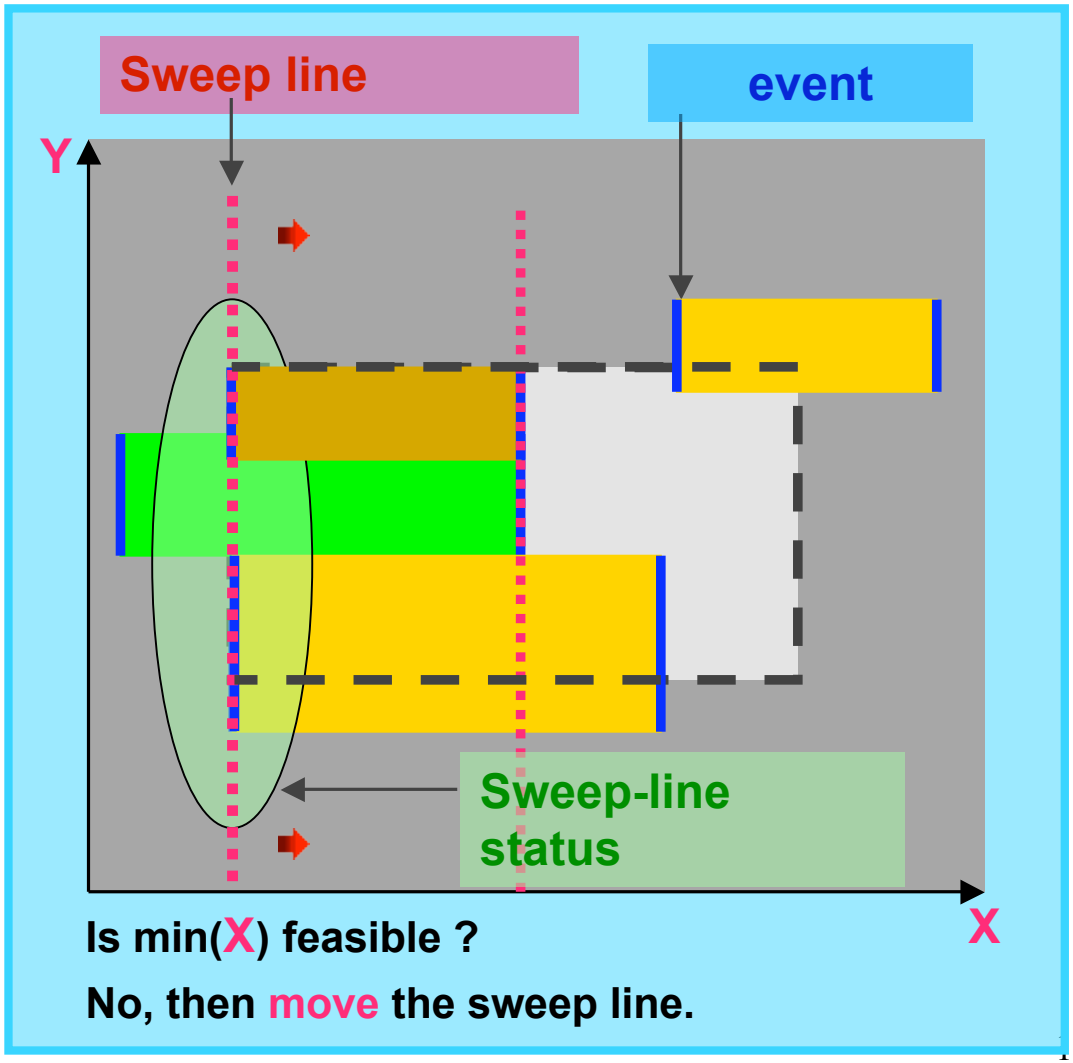
  [Boissonnat & Yvinec, 1995]

Basic Idea of the Sweep Algorithm
(in dimension 2)

Accumulates forbidden regions

$$\begin{cases} CTR_1(X,Y,\ldots) \\ CTR_2(X,Y,\ldots) \\ \ldots\ldots\ldots\ldots\ldots \\ CTR_n(X,Y,\ldots) \end{cases}$$

sharing 2 given variables X and Y



Question: How to Generalize to *k* Dimensions ?

Key problem with the sweep-line status:

don't want to use a multi-dimensional data structure since it just kills scalability

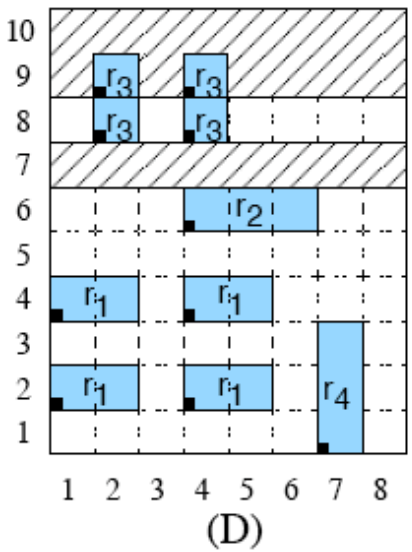
Geometric Kernel : a Lexicographic Sweep-Point Algorithm

VARIABLES
x1 in 1..4, y1 in 2..4
x2 in 4..4, y2 in 6..6
x3 in 2..4, y3 in 8..9
x4 in 7..7, y4 in 1..1
x5 in 1..8, y5 in 1..8, y5<>7

EXTERNAL CONSTRAINT (non–overlapping)
geost([object(1,1,[x1,y1],0,1,1),object(2,2,[x2,y2],0,1,1),
object(3,3,[x3,y3],0,1,1),object(4,4,[x4,y4]0,1,1),
object(5,5,[x5,y5],0,1,1)],
[shape(1,[0,2],[0,1]),shape(2,[0,3],[0,1]),shape(3,[0,1],[0,1]),
shape(4,[0,1],[0,3]),shape(5,[0,5],[0,4])],
[non–overlapping([0,1],[1,2,3,4,5])])

INTERNAL CONSTRAINTS GENERATED FOR FILTERING THE ORIGIN OF THE FIFTH OBJECT, i.e. (x5,y5) (ICTRS)
ctr1: outbox([1,1],[2,2]) ctr3: outbox([1,8],[2,1])
ctr2: outbox([1,3],[6,4]) ctr4: outbox([3,1],[5,3])
ctr5: outbox([1,7],[8,1])

(A)

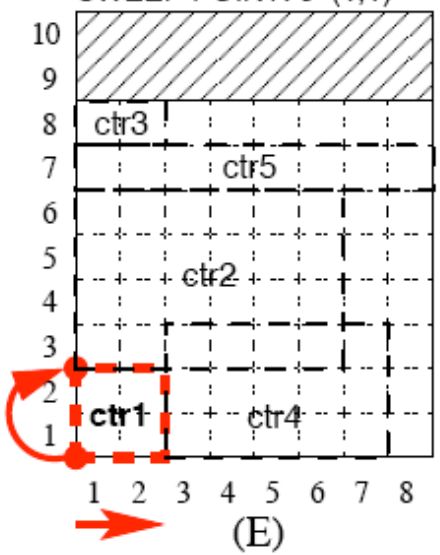


(D)

DCTRS
(delayed internal constraint)
ctr1
ctr2
ctr5
ctr3
ctr4

ACTRS
(active internal constraint)

(B)



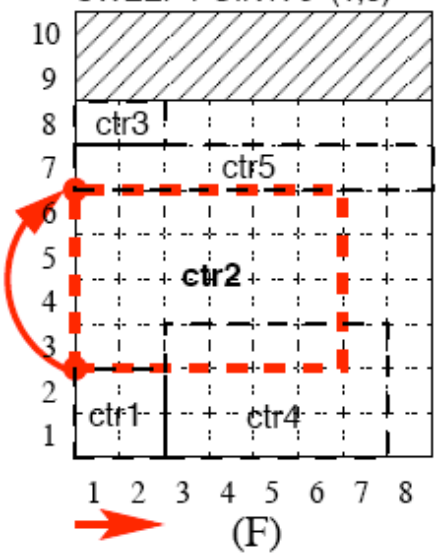
(E)

DCTRS
ctr2
ctr5
ctr3
ctr4

ACTRS
ctr1

CONFLICT
ctr1

(C)



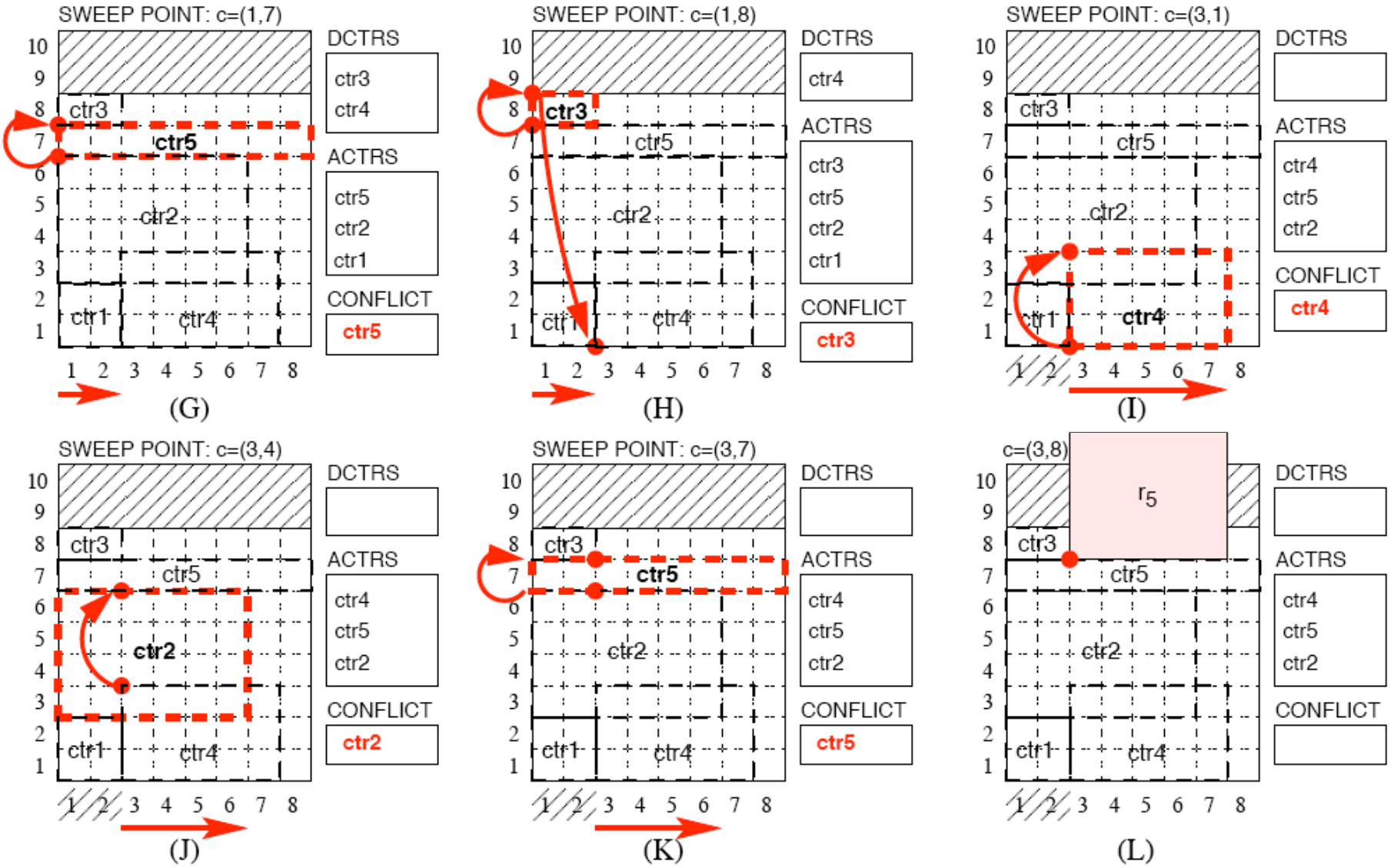
(F)

DCTRS
ctr5
ctr3
ctr4

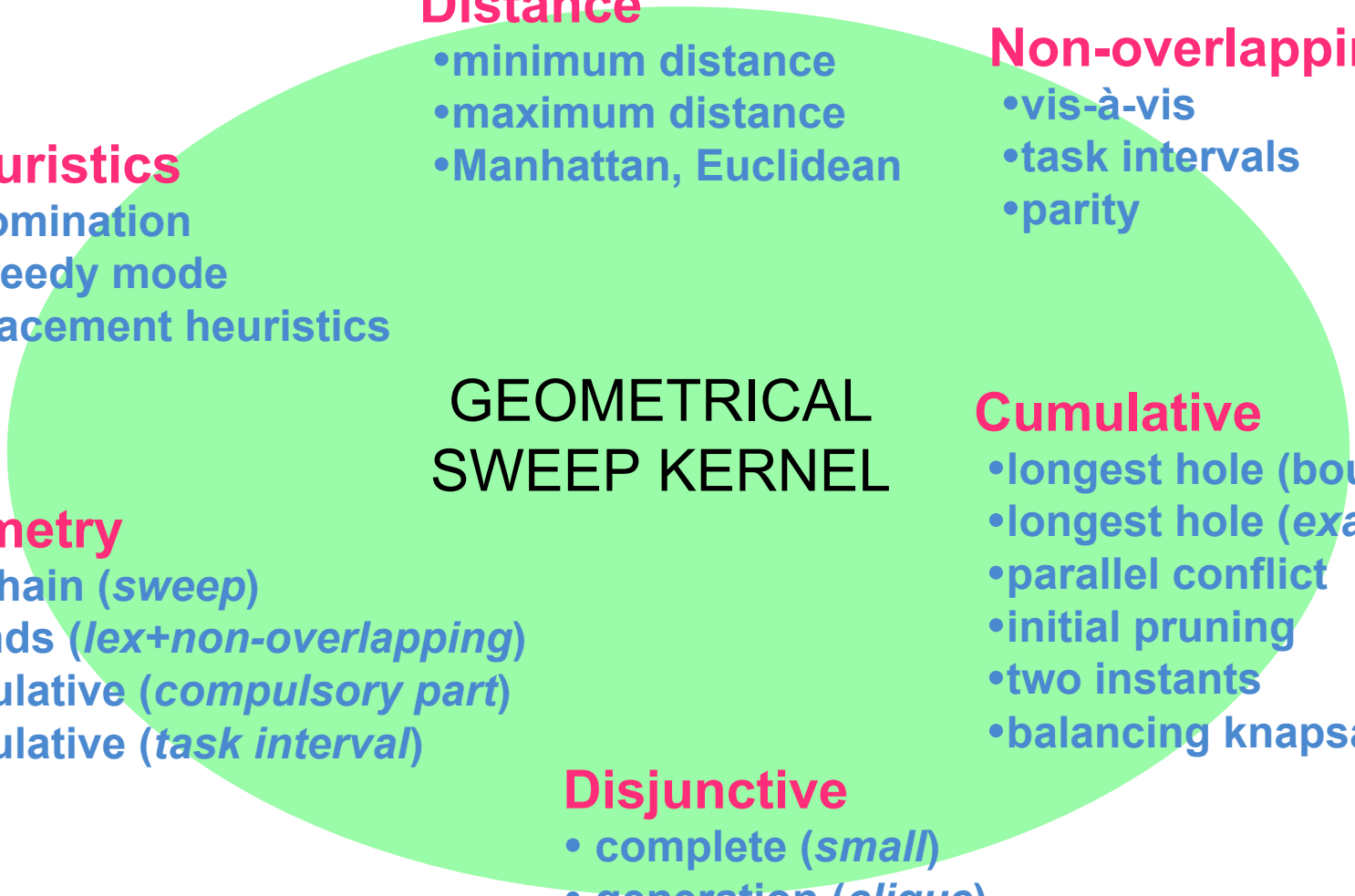
ACTRS
ctr2
ctr1

CONFLICT
ctr2

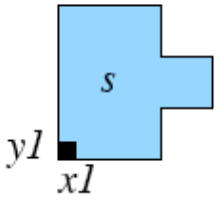
Geometric Kernel : a Lexicographic Sweep-Point Algorithm



Extensions Around the Kernel

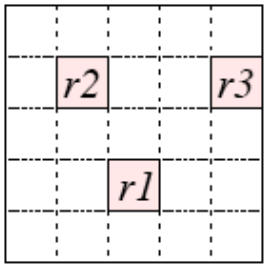


Where Splitting Objects Kills Propagation



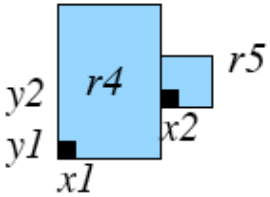
Shape to place within (B) so that it does not overlap rectangles r1, r2, r3

(A)



Placement space where to put s

(B)



Decomposing s in two rectangles r4 and r5

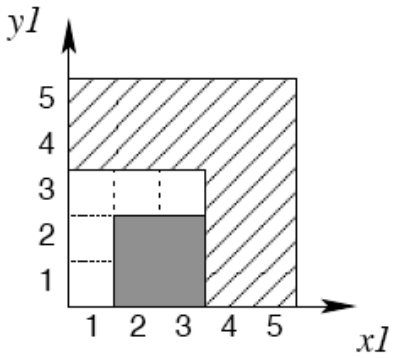
(C)

ctr1: x1+2=x2
ctr2: y1+1=y2
ctr3: r1 and r4 do not overlap
ctr4: r2 and r4 do not overlap
ctr5: r3 and r4 do not overlap
ctr6: r1 and r5 do not overlap
ctr7: r2 and r5 do not overlap
ctr8: r3 and r5 do not overlap

Constraints of the problem

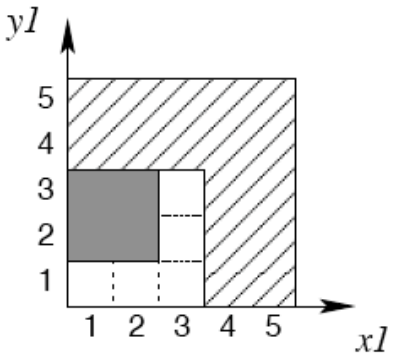
(D)

Where Splitting Objects Kills Propagation



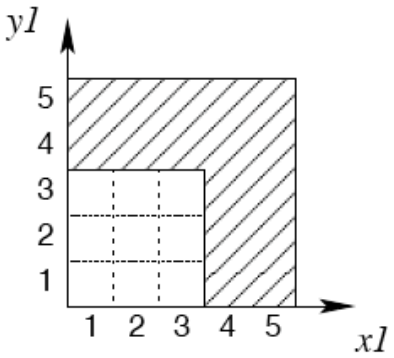
Forbidden pairs of values for (x1,y1) according to ctr3

(E)



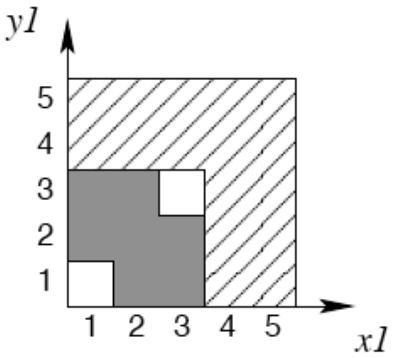
Forbidden pairs of values for (x1,y1) according to ctr4

(F)



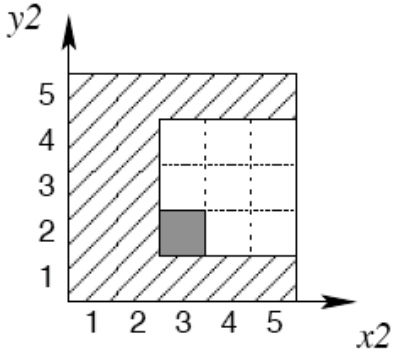
Forbidden pairs of values for (x1,y1) according to ctr5

(G)



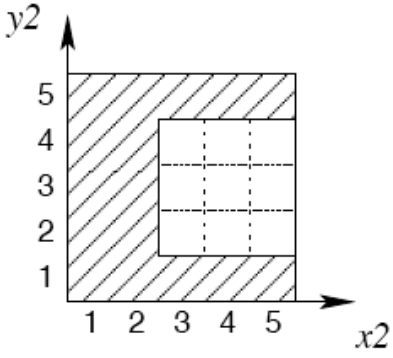
Forbidden pairs of values for (x1,y1) according to ctr3, ctr4 and ctr5

(H)



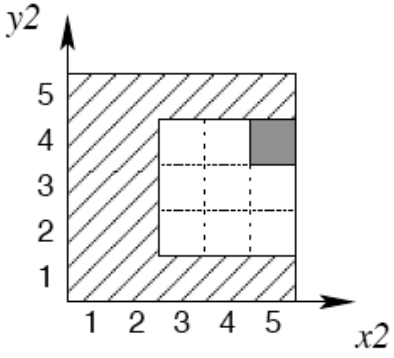
Forbidden pairs of values for (x2,y2) according to ctr6

(I)



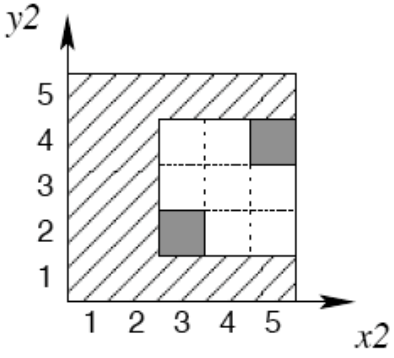
Forbidden pairs of values for (x2,y2) according to ctr7

(J)



Forbidden pairs of values for (x2,y2) according to ctr8

(K)

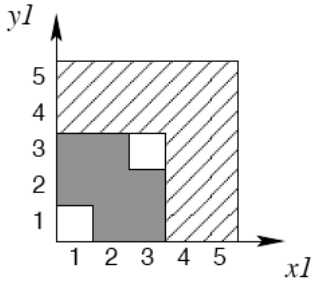


Forbidden pairs of values for (x2,y2) according to ctr6, ctr7 and ctr8

(L)

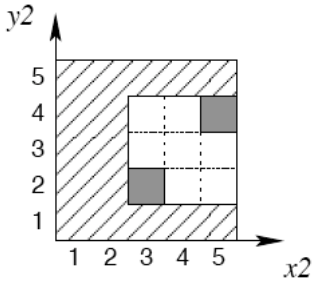
Where Splitting Objects Kills Propagation

QUESTION :
How to combine information from (x1,y1) and (x2,y2) ?



Forbidden pairs of values for (x1,y1) according to ctr3, ctr4 and ctr5

(H)

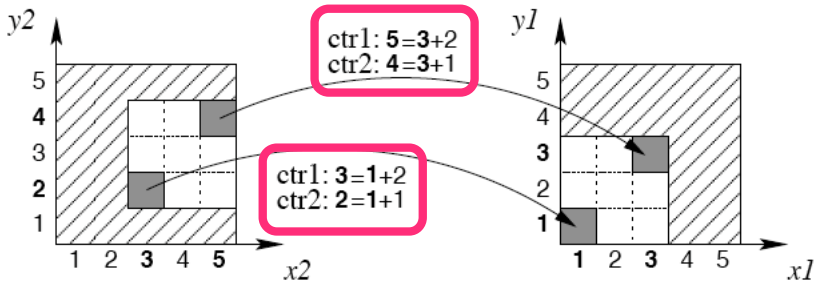


Forbidden pairs of values for (x2,y2) according to ctr6, ctr7 and ctr8

(L)

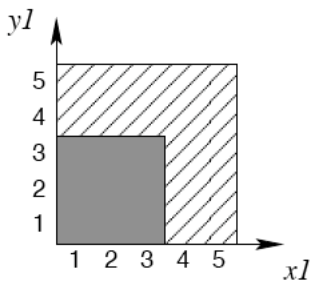
ctr1: x1+2=x2
ctr2: y1+1=y2

ANSWER :
Combining the infeasible points for (x1,y1) and (x2,y2) ONLY possible if ctr1 and ctr2 are integrated within the sweep process !



Transmission of the forbidden pairs of values for (x2,y2) to forbidden pairs of values for (x1,y1) through the external constraints ctr1 and ctr2 is not done if ctr1 and ctr2 are not integrated within the sweep algorithm

(M)



Overall set of forbidden pair of values for (x1,y1)

(N)

Introduction

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geost on the web

A First Evaluation
with a focus on non-overlapping

- **Scalability** on **loosely constrained problems** (20% spare space)

- **Tight** placement problems (**0% spare space**)
  - Perfect squared squares
  - 3D pentominoes [Colmerauer, Gilleta 99]

- **State of the art OR** for **2D orthogonal packing** [Clautiaux, Carlier, Jouglet 07]

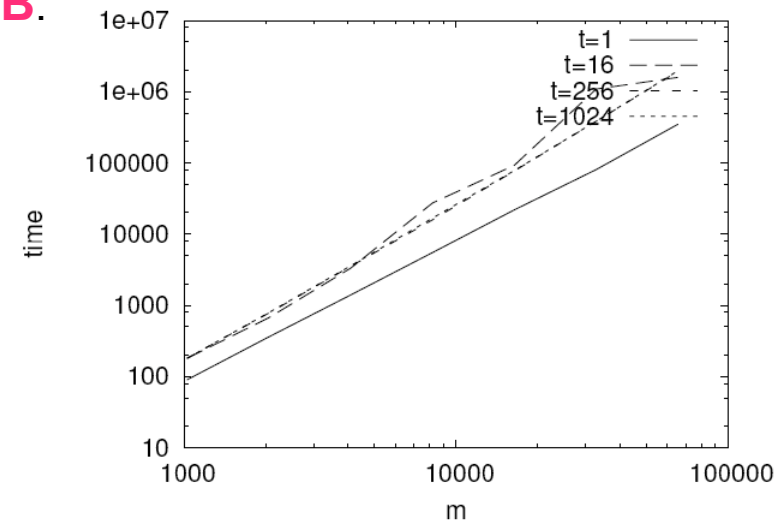
Loosely Constrained Problems

- Search first solution for random problem instances of **m** **k**-dimensional boxes for **k** in {2,3,4} involving **t** in {1,16,256,1024} distinct types of boxes, and **m** in {1024,2048,…,65536}.

- The number of **1.048.576** variables in *geost* was reached (***first time in a constraint solver in a backtracking environment !***).

- Can typically pack **1024 2D, 3D and 4D distinct boxes** in at most **200 msec**.
- Worst time, **13694 sec**, obtained for packing **262.144 4D parallelepipeds** corresponding to **1024 distinct types**, with a memory consumption of **351MB**.
- Approach sensible to the number of distinct types of boxes.

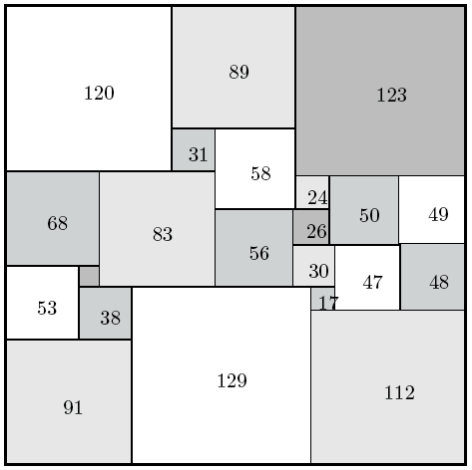
Results for **k**=2 for various **t** and **m** (time in msec)



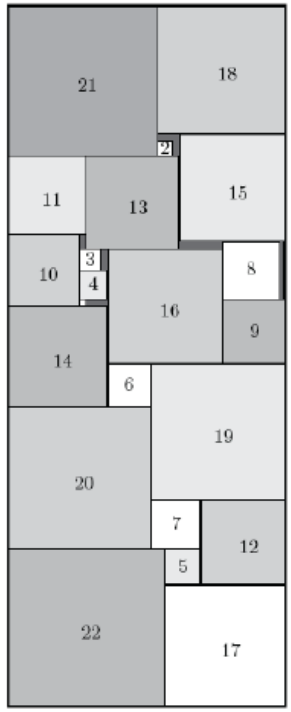
ON THE WAY: **MEMOIZATION** SHOULD ENHANCE THESE RESULTS WHEN NOT TOO MANY DISTINCT SIZES !

Tight Placement Problems

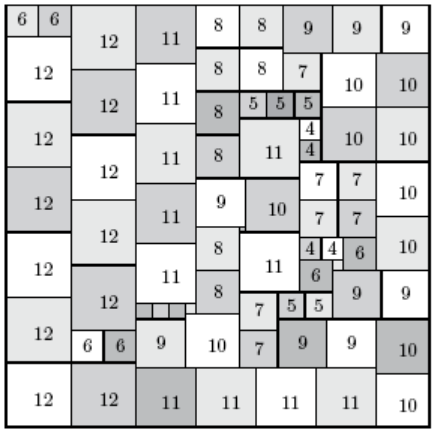
Squared squares



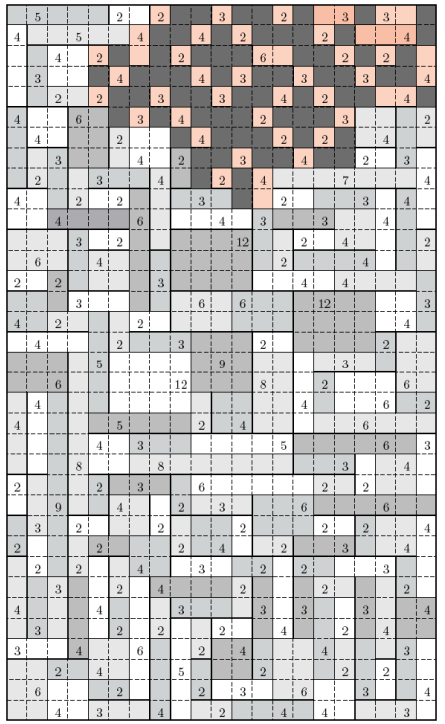
Smallest surface



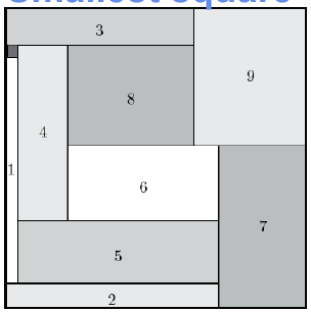
Partridge



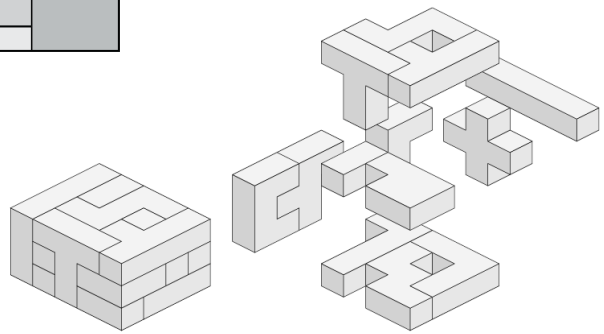
Shikaku



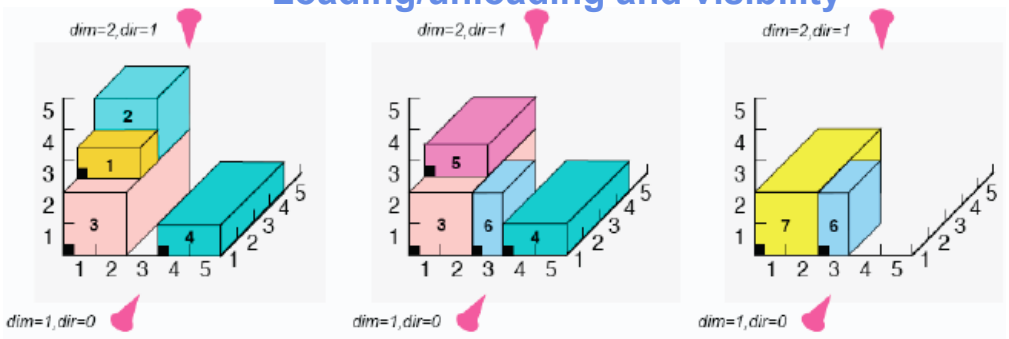
Smallest square



3-D pentomino (Giletta)



Loading/unloading and visibility



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geost on the web

Handling time

Motivation: a lot of practical problems mix geometrical constraints with time:

(1) Constructing a 3D packing,
(2) Pick-up delivery (objects stays between two dates),
(3) Containers management in a harbour (*containers are organised in piles and stay between two dates*),
(4) Planning trajectory (several consecutive moves for one object).

Extending *geost*

- **Solution**: add 3 (or 2) extra dimensions (*origin*,*duration*,*end*) to the sweep algorithm.
- **Warning**: when the duration is variable the origin and end attributes are not equivalent !
- **Warning**: special care if one wants to generate the exact forbidden region with respect to the constraint *origin*+*duration*=*end* (using boxes kills the propagation since too many boxes).
- Trajectory made up from several objects: handle also **time** and **space continuity** constraints as well as **velocity**.

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***geost* on the web**

geost on the web

- Integrated by **EMN** and **SICS** within **CHOCO** and **SICStus**
- since Nov. 2008 within the **Global Constraints Catalog**
  see http://www.emn.fr/x-info/sdemasse/gccat/

(1) description of *geost* (*and other geometric constraints*)

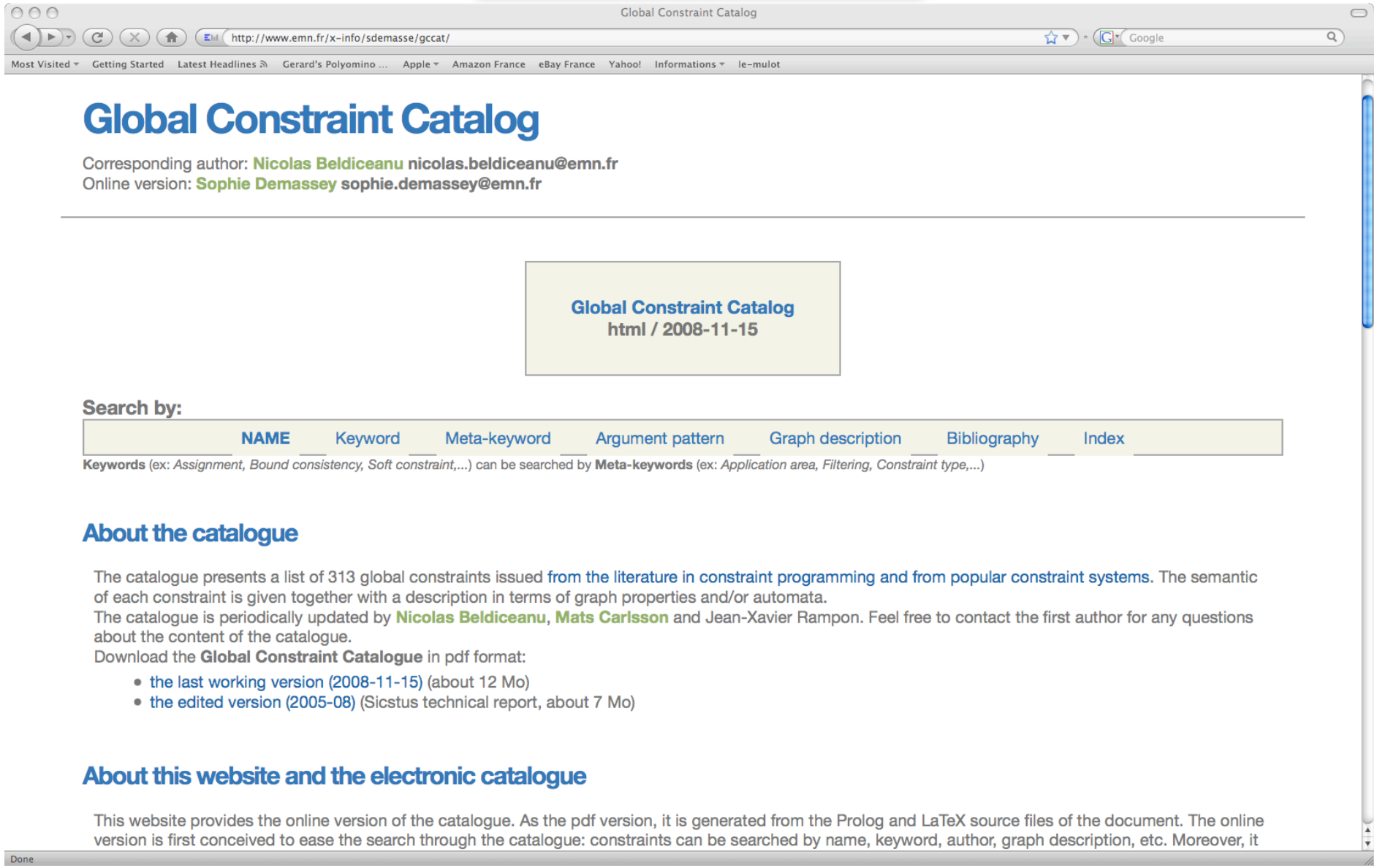
(2) overview (*and pointers*) to **propagation** techniques

(3) overview and illustration of placement **heuristics**

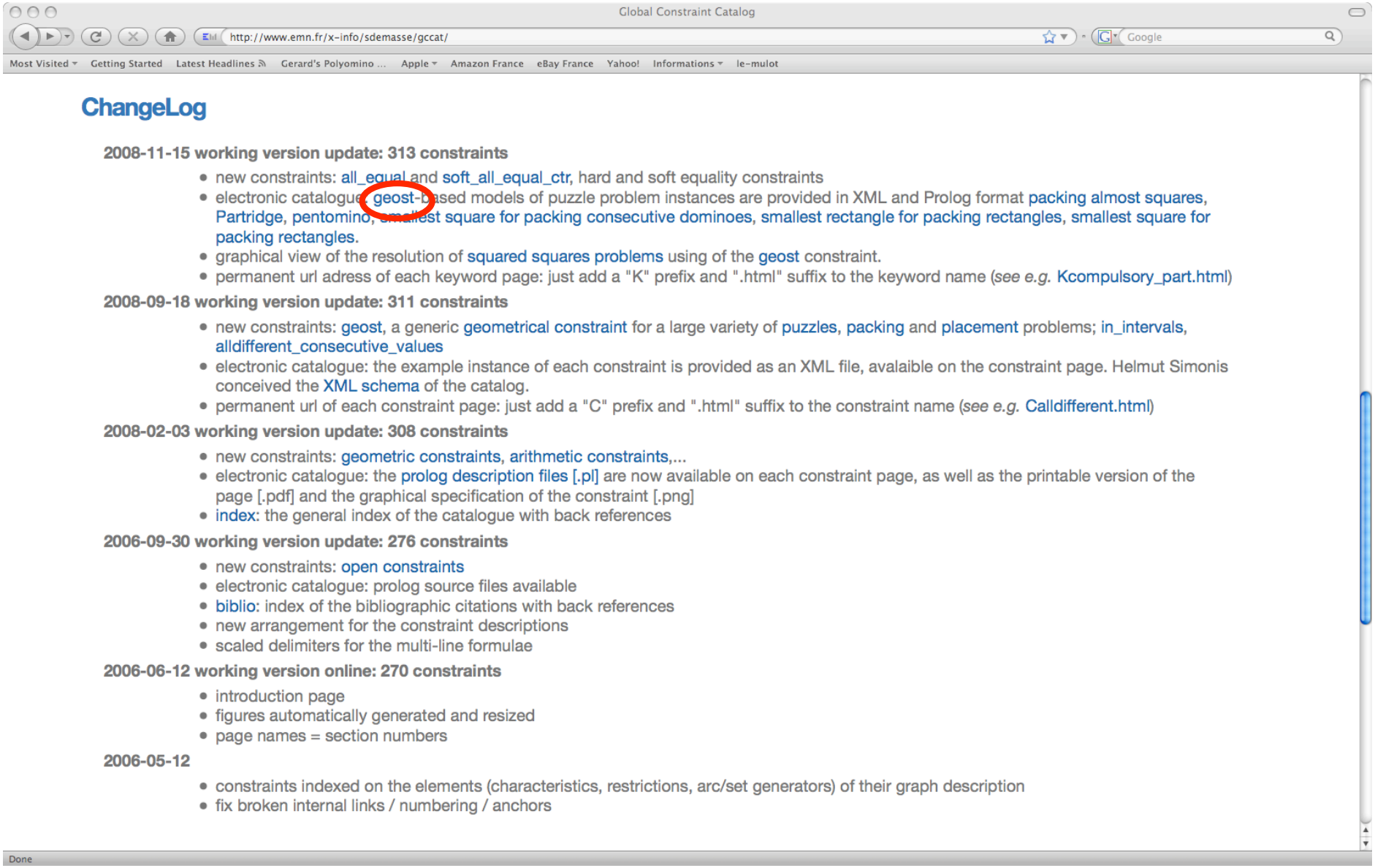
(4) overview of placement **problems**

(5) **xml** format for *geost* and problem instances

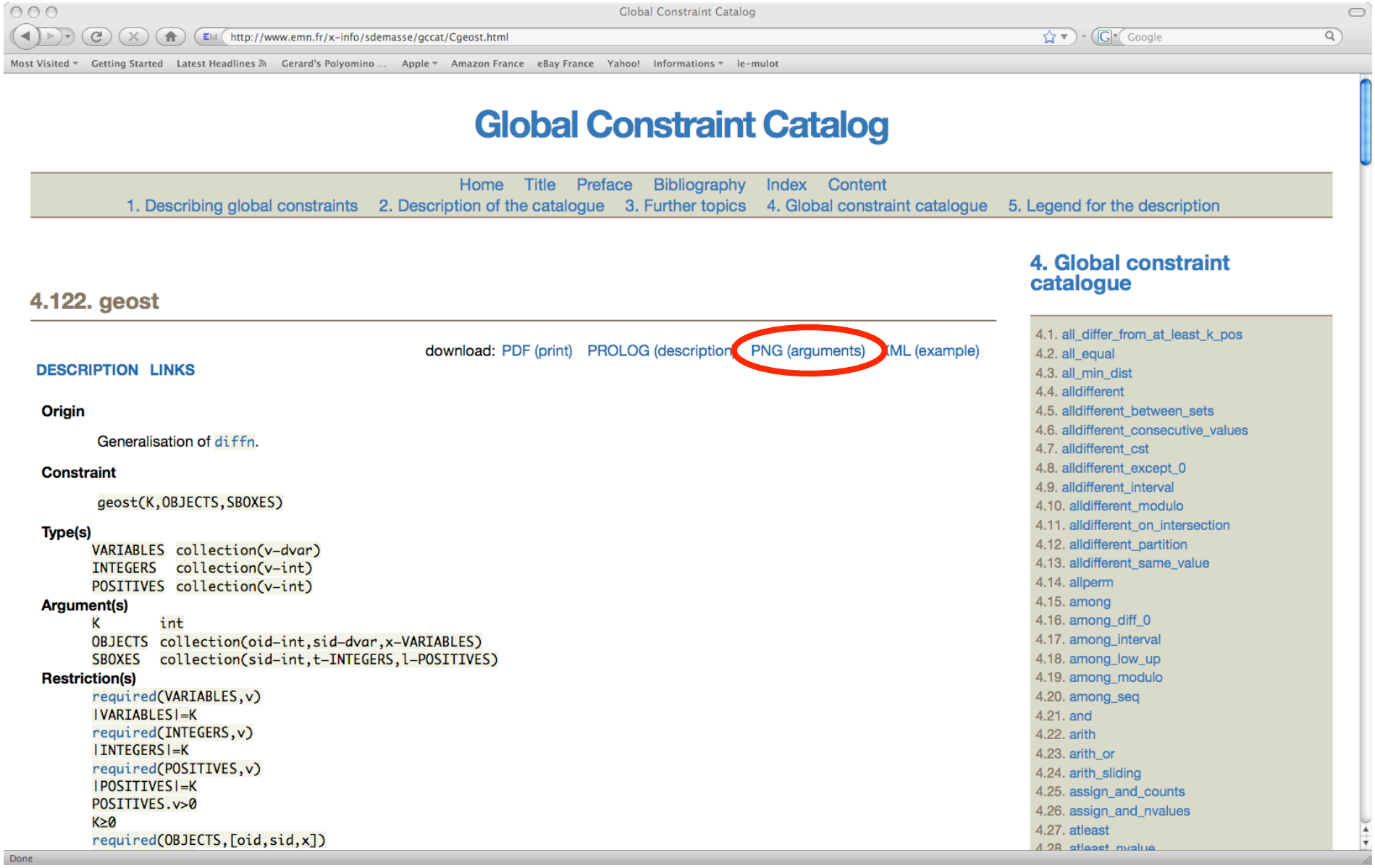
geost on the web



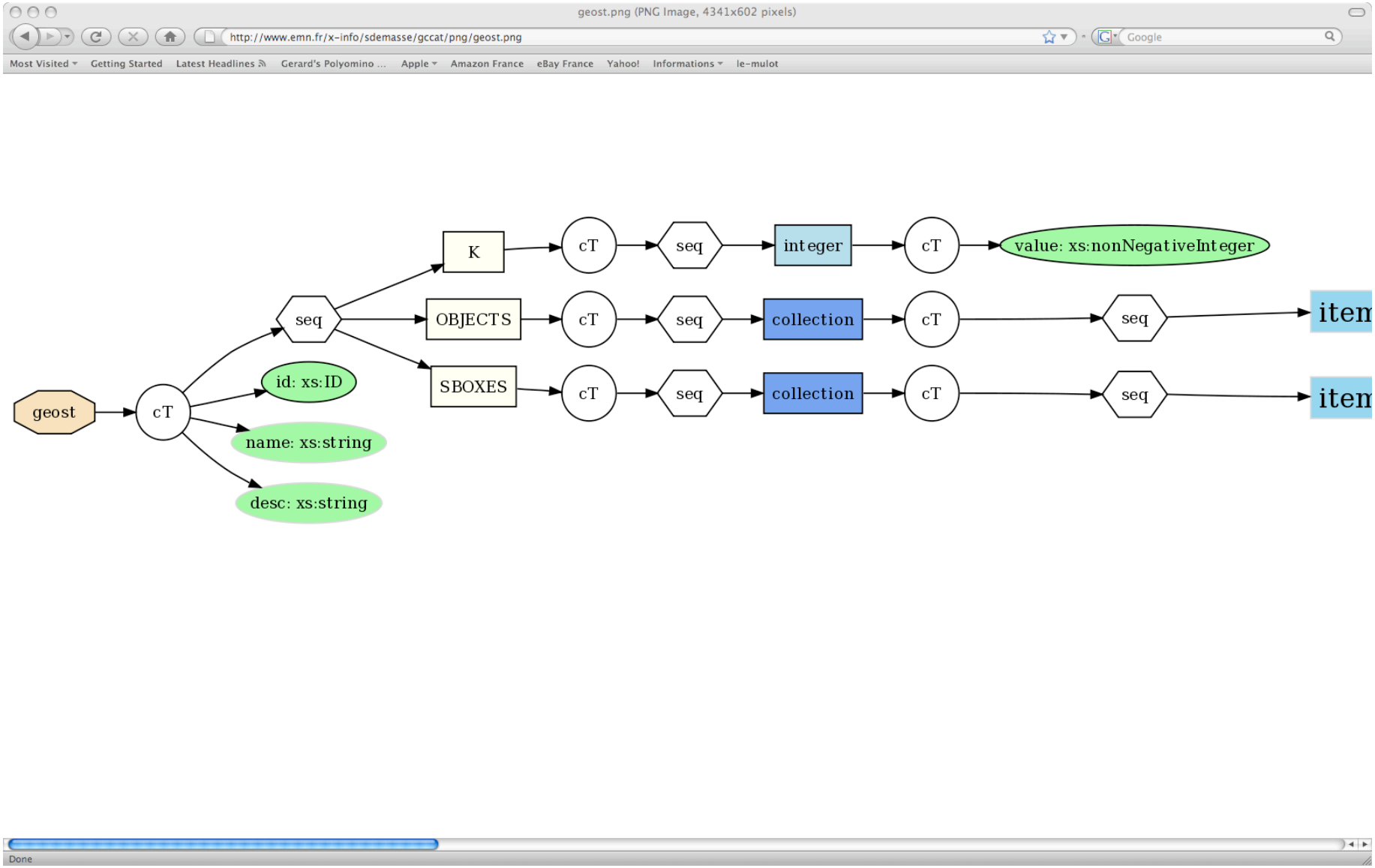
geost on the web



geost on the web



geost on the web



Conclusion

Once again, use the **sweep** idea: *quite simple, but powerful !*

- The overall architecture was designed in order to allow to integrate additional constraints **without modifying** the kernel.
- Can directly handle objects that **move in time**.
- When propagating on one object, consider **all constraints** involving that object.
- Scale better (**one million integer variables** in a standard constraint system: compatible with **backtracking**).

One last observation:

disjunctive, cumulative, non-overlapping constraints
*should all be integrated within **one single global constraint** since:*
(1) they all correspond to related nested dynamic sub problems,
(2) allow to get better propagation,
(3) allow to reuse code.

Publications about *geost* EMN-INRIA-SICS

- **N. Beldiceanu**, **M. Carlsson**, **E. Poder**, **R. Sadek**. *A Generic Geometrical Constraint Kernel in Space and Time for Handling Polymorphic k-Dimensional Objects*. In C. Bessiere, editor, Principles and Practice of Constraint Programming (**CP'2007**), vol. 4741 of LNCS, pages 180-194, Springer 2007.

- **N. Beldiceanu**, **M. Carlsson**, **E. Poder**. *New Filtering for the cumulative Constraint in the Context of Non-Overlapping Rectangles*. In L. Perron and M. Trick editor, Integration of AI and OR techniques in Constraint Programming for Combinatorial Optimization Problems (**CPAIOR'2008**), vol. 5015 of LNCS, pages 21-35, Springer 2008.
  JOURNAL VERSION SUBMITTED

- **M. Carlsson**, **N. Beldiceanu**, **J. Martin**. *A Geometric Constraint over k-dimensional Objects Subject to Business Rules*. In P. Stuckey, editor, Principles and Practice of Constraint Programming (**CP'2008**), vol. 5202 of LNCS, pages 220-234, Springer 2008.
  INVITED FOR A SPECIAL ISSUE OF THE Constraint JOURNAL