Fundamentals of Integer Programming

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January 2025

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Outline

- Definition of integer programming
- Formulating some classical problems with integer programming
- Linear programming
- Solution methods for integer programming
- Solvers and their interface
- Impact of modeling on problem solving

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Optimization and Programming

- Mathematical programming: to find the best solution from a set of alternatives
- Mathematical models for optimization problems
  - Optimization variables encoding problem solution
  - Constraints defined using **mathematical functions** and **equation/inequality**
  - Objective function (a mathematical function of the variables)

$$\frac{\max}{\min} \quad f(\boldsymbol{x})$$
$$\text{s. t.} \quad g_i(\boldsymbol{x}) \sim b_i, i = 1, \ldots, m$$

$\boldsymbol{x}$ is the vector of variables, and $\sim$ can be any of $\leq$, $=$, and $\geq$

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Types of Optimization Models

$$\frac{\max}{\min} \quad f(\boldsymbol{x})$$
$$\text{s. t.} \quad g_i(\boldsymbol{x}) \sim b_i, i = 1, \ldots, m$$

$$\frac{\max}{\min} \quad \sum_{j=1}^{n} c_j x_j$$
$$\text{s. t.} \quad \sum_{j=1}^{n} a_{ij} x_j \sim b_i, i = 1, \ldots, m$$
$$x_j \text{ integer}, j = 1, \ldots, k \text{ (where } k \leq n)$$

- Linear functions + continuous variables: linear programming
- Linear functions + integer variables: integer (linear) programming
  - Mixed integer programming: Both integer and continuous variables
  - Special case: Binary variables
- (Nonlinear programming and integer nonlinear programming)

Note: All combinatorial optimization problems can be formulated as (mixed) integer programming models

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More on Mathematical Programming Models

- A mathematical programming model always uses mathematical functions to define constraints
- Examples of statements that do not qualify
  - x != y
  - If x > 0 then y = 1 (assuming y is binary; either 0 or 1)
  - Either x or y must be zero
  - max(x, y) >= 1
- For combinatorial optimization, in many/most cases we can translate such conditions using functions and equality/inequality
  - x <= M y, where M is a big enough number, will ensure y = 1 if x > 0
  - x y = 0 implies at least one of the two must be zero (even though x y is a nonlinear function and hence not easy to deal with)

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Binary Knapsack

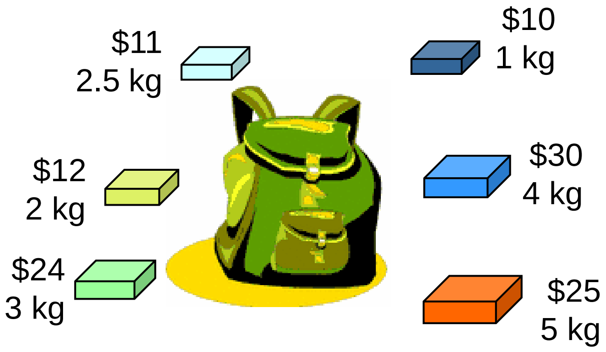
- Given:
  - A set of n items, each with a value c_i and weight a_i
  - A knapsack with a weight limit b
- Select items to maximize the total value of the knapsack, without exceeding the weight limit

$x_i = 1$ if item i is chosen

$$\max \quad \sum_{i=1}^{n} c_i x_i$$

$$\text{s. t.} \quad \sum_{i=1}^{n} a_i x_i \leq b$$

$$\boldsymbol{x} \in \{0,1\}$$



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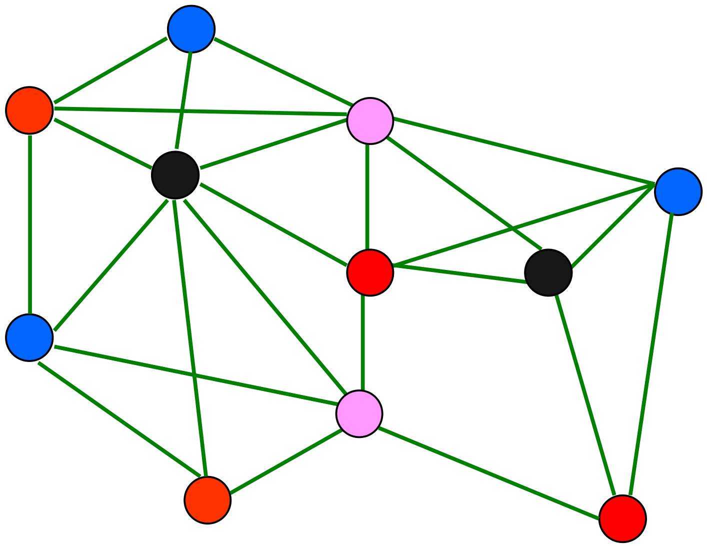
Coloring

- Given: a graph with nodes and edges
- Assign a color to each vertex (node); two adjacent vertexes must use different colors
- Minimize the total number of colors used



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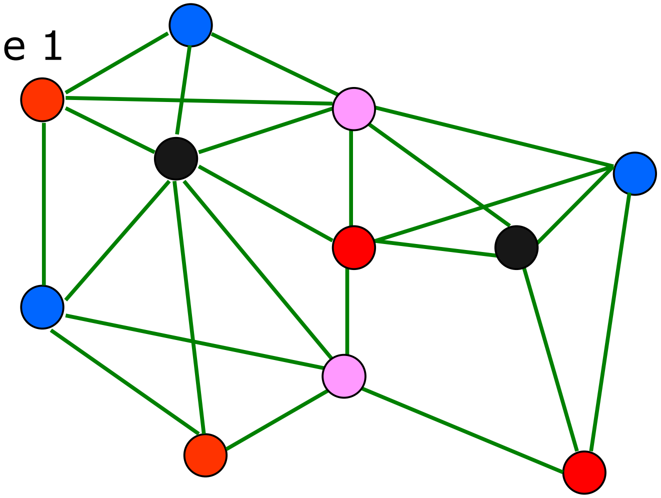
Coloring (cont'd)

z_i = color (an integer value) of node i

$$\begin{aligned} \min \quad & \max_i z_i \\ \text{s. t.} \quad & z_1 \neq z_2 \\ & \dots \\ & \boldsymbol{z} \ \text{ integer} \end{aligned}$$



However, as was mentioned, an integer programming model cannot deal with a constraint like $z_1 \neq z_2$, or the case of an "or" condition: $z_1 \geq z_2 + 1$ or $z_1 \leq z_2 - 1$

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Coloring (cont'd)

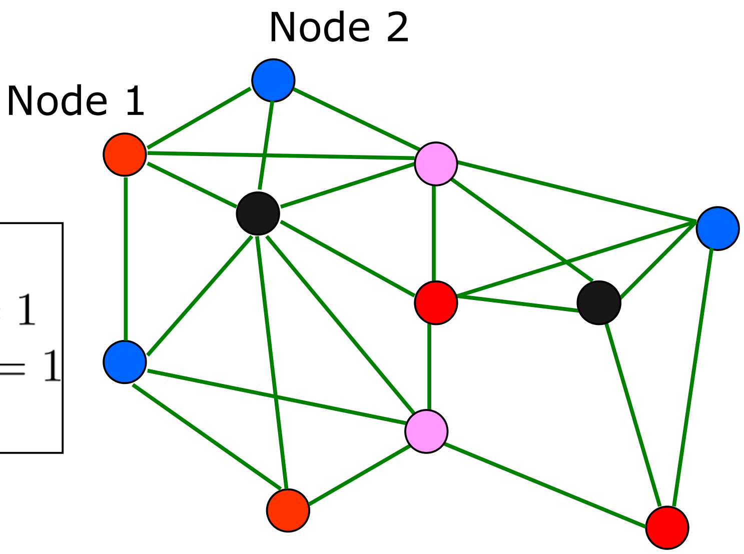
Define color set C

- $x_{ic} = 1$ if node i has color c
- $y_c = 1$ if color c is used (by some node)

$$\begin{aligned}
\min \quad & y_{red} + y_{blue} + \dots \\
\text{s. t.} \quad & x_{1,red} + x_{1,blue} + \dots = 1 \\
& x_{2,red} + x_{2,blue} + \dots = 1 \\
& \dots
\end{aligned}$$

$$\boxed{\begin{array}{ll} x_{1,red} \leq y_{red} & \text{Formulates} \\ x_{1,blue} \leq y_{blue} & x_{1,red} = 1 \to y_{red} = 1 \\ \dots & x_{1,blue} = 1 \to y_{blue} = 1 \end{array}}$$

$$\begin{aligned}
& x_{1,red} + x_{2,red} \leq 1 \\
& x_{1,blue} + x_{2,blue} \leq 1 \\
& \dots \\
& \boldsymbol{x} \in \{0,1\}, \boldsymbol{y} \in \{0,1\}
\end{aligned}$$



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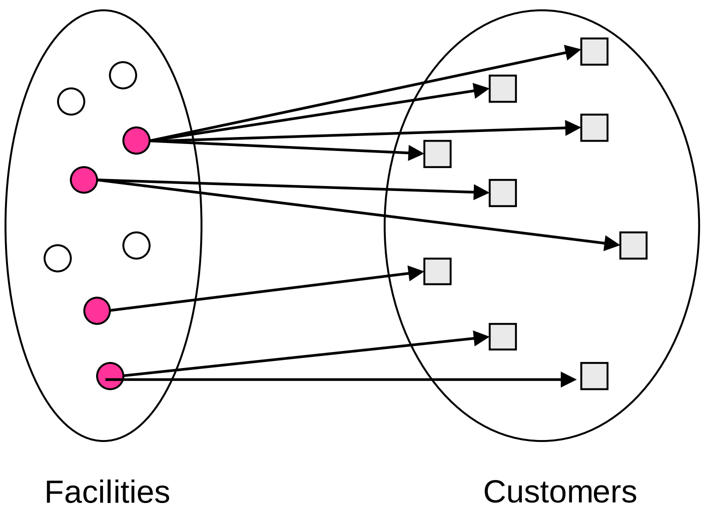
Uncapacitated Facility Location

- Given:
  - A set of candidate facility (e.g., warehouse) locations
  - A set of customers
  - Opening a facility has a fixed charge
  - Transportation cost between facility locations and customers
- Determine which facilities to deploy and the customers served by each deployed facility, minimizing the total cost



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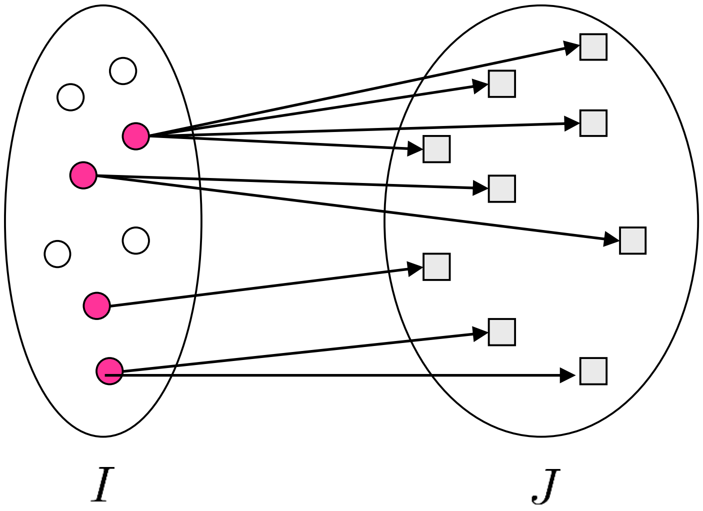
Uncapacitated Facility Location (cont’d)

- I: Candidate set of facility locations
- J: Set of customers
- f_i: Fixed charge, $i \in I$
- c_{ij}: Transportation cost, $i \in I, j \in J$

- $x_{ij} = 1$ if facility $i \in I$ serves customer j
- $y_i = 1$ if facility $i \in I$ is deployed

$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \\
\text{s. t.} \quad & \sum_{i \in I} x_{ij} = 1, \forall j \in J \\
& x_{ij} \le y_i, \forall i \in I, \forall j \in J \\
& \boldsymbol{x}, \boldsymbol{y} \in \{0, 1\}
\end{aligned}
$$



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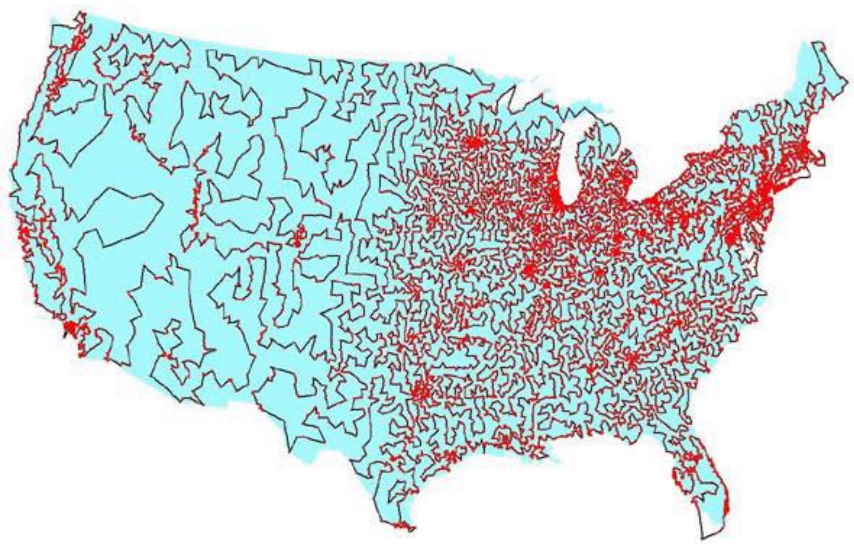
Traveling Salesman Problem

- Given: a graph with edge costs
- Find a tour visiting each node in a graph exactly once with minimum length



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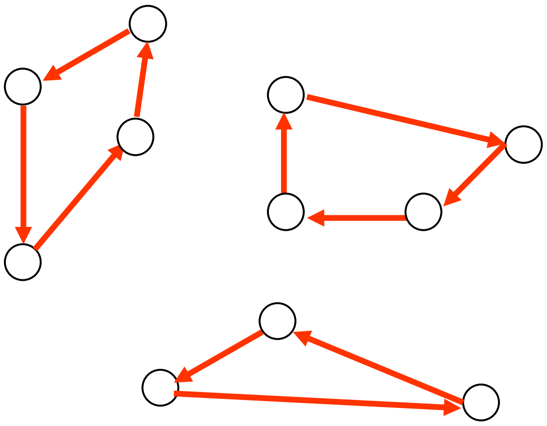
Traveling Salesman Problem (cont'd)

- N: Set of nodes
- c_{ij}: Cost of edge (i, j)

- How to formulate this problem by integer programming?

- $x_{ij} = 1$ if city j is visited immediately after city i $(i \neq j)$

$$\begin{aligned} \min \quad & \sum_{i \in N} \sum_{j \in N: j \neq i} c_{ij} x_{ij} \\ \text{s. t.} \quad & \sum_{j \in N: j \neq i} x_{ij} = 1, \forall i \in N \\ & \sum_{j \in N: j \neq i} x_{ji} = 1, \forall i \in N \\ & \boldsymbol{x} \in \{0, 1\} \end{aligned}$$



Have we overlooked anything?

$$\sum_{i \in S} \sum_{j \in N \setminus S} x_{ij} \geq 1, \forall S \subset N$$

Potential drawback of the formulation?

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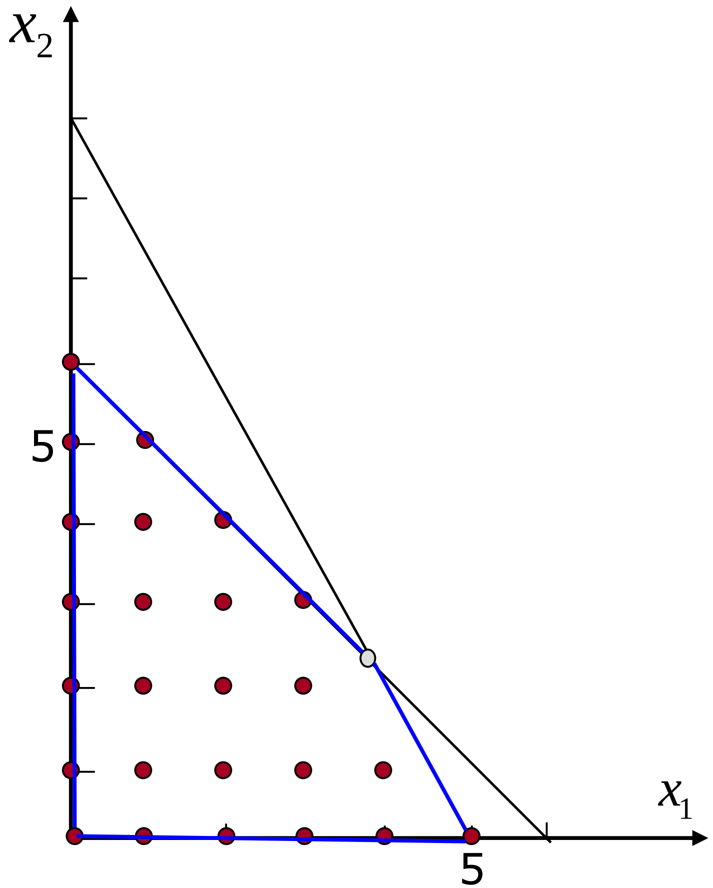
A Small Example

$$\begin{aligned} \max \quad & 8x_1 + 5x_2 \\ \text{s. t.} \quad & x_1 + x_2 \leq 6 \\ & 9x_1 + 5x_2 \leq 45 \\ & x_1 \geq 0, x_2 \geq 0, \text{integer} \end{aligned}$$

$$\begin{aligned} \max \quad & 8x_1 + 5x_2 \\ \text{s. t.} \quad & x_1 + x_2 \leq 6 \\ & 9x_1 + 5x_2 \leq 45 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$



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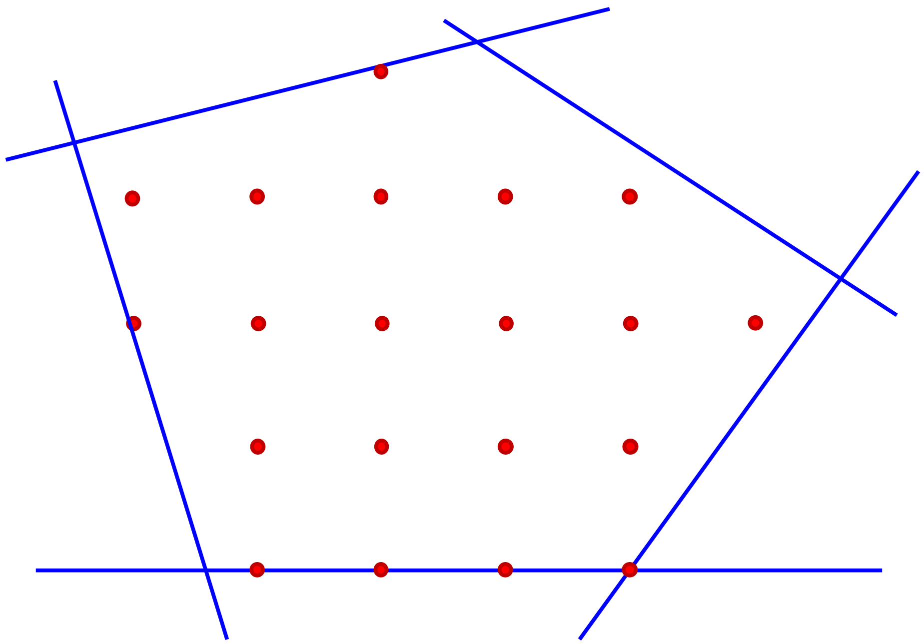
Linear Programming Relaxation

- Relaxation: “removal” of some constraints/restrictions
- In general, the linear programming relaxation is an approximation of the integer model; the solution of the former may be fractional
- Which one is easier to solve?
- LP is in P



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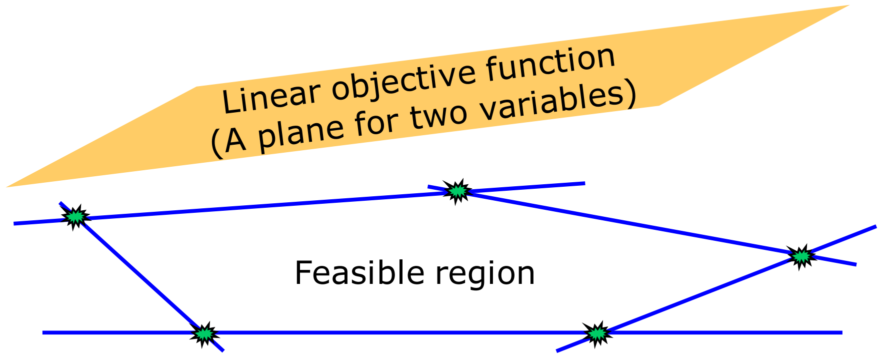
Solving a Linear Programming Model

- Fundamental property: Optimum is located at one of the extreme/corner points of the feasible region (why?)



- This is used by the Simplex Method for solving linear programs (visiting a sequence of objective-improving extreme points)
- There are other efficient, interior-point methods

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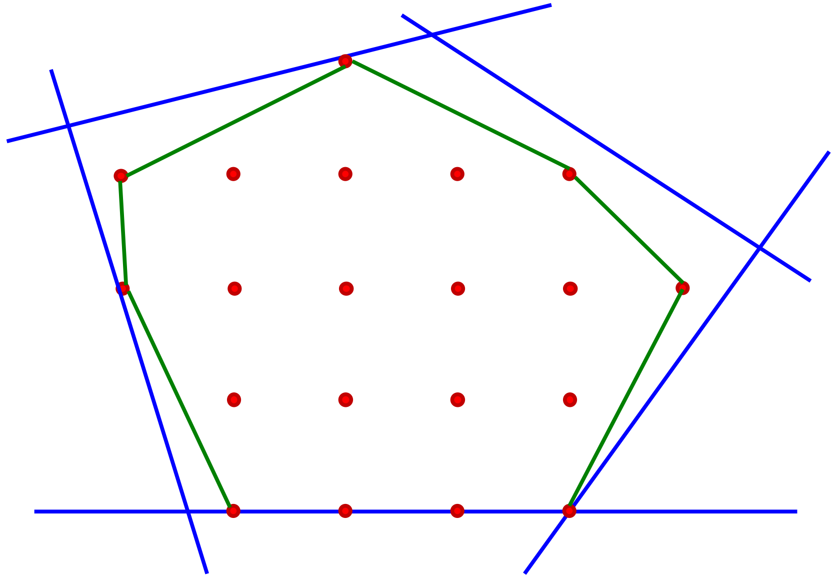
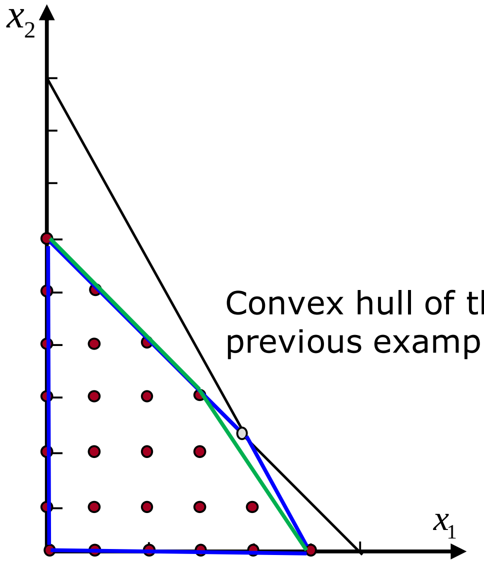
The Convex Hull

- Convex hull: The minimum convex set containing the solution space





- Integer programming = linear programming on the convex hull of the integer points
- Convex hull exists, but its description is hard to derive in general

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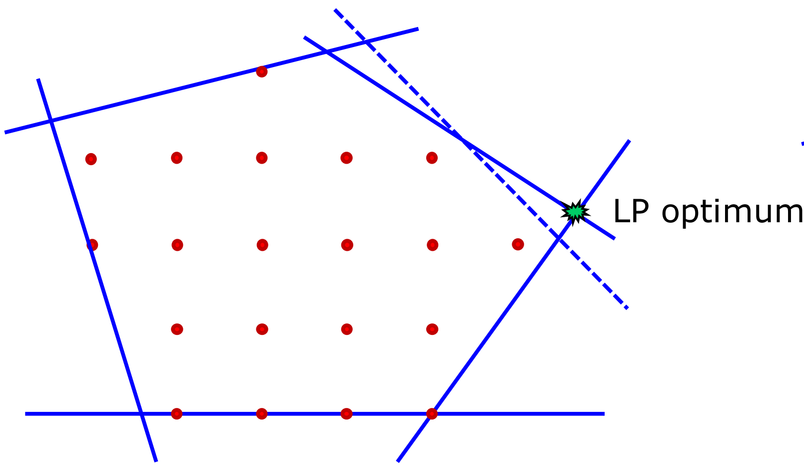
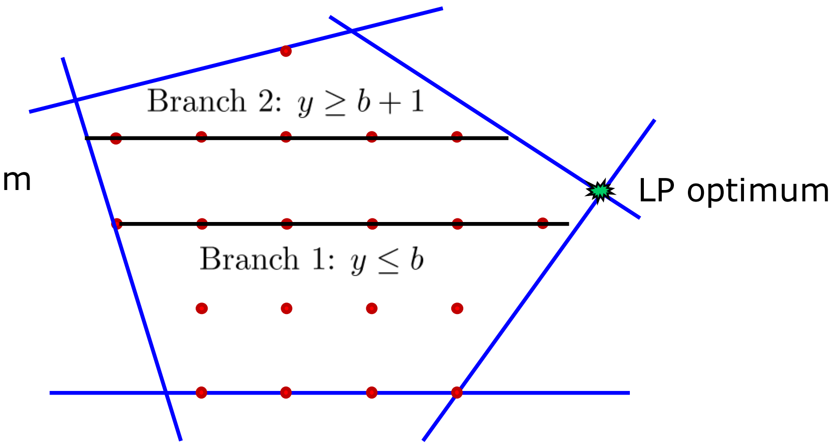
Computing the Global Optimum

- General-purpose method: Linear programming relaxation +
  - Iterative improvement in approximating the convex hull, a.k.a. **cutting planes** (cf. inference)
  - Divide-and-conquer, a.k.a. **branch-and-bound** (relaxation + search)





- **Optimality gap**: The (relative) difference between the objective value of the best known integer solution and that of the best (“optimistic”) LP bound so far

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Cutting Planes: An Example

Knapsack problem instance:

$$\begin{aligned} \max \quad & 6x_1 + 4x_2 + 6x_3 + 7x_4 + 5x_5 + 9x_6 + 8x_7 \\ \text{s. t.} \quad & 5x_1 + 6x_2 + 8x_3 + 6x_4 + 4x_5 + 6x_6 + 5x_7 \leq 21 \\ & \boldsymbol{x} \in \{0, 1\} \end{aligned}$$

LP optimum: $x_1 = x_5 = x_6 = x_7 = 1,\ x_4 = 0.167,\ x_2 = x_3 = 0$

Can we pack items 1, 4, 6, and 7 all in the knapsack? (5+6+6+5=22)

$$\Rightarrow x_1 + x_4 + x_6 + x_7 \leq 3$$

The above inequality (referred to as a “cover cut”) is valid for integer solutions, but violated by the LP relaxation optimum

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Cutting Planes: An Example (cont'd)

Adding the cut to the linear programming relaxation:

$$\begin{array}{rl} \max & 6x_1 + 4x_2 + 6x_3 + 7x_4 + 5x_5 + 9x_6 + 8x_7 \\ \text{s. t.} & 5x_1 + 6x_2 + 8x_3 + 6x_4 + 4x_5 + 6x_6 + 5x_7 \leq 21 \\ & x_1 + x_4 + x_6 + x_7 \leq 3 \\ & \mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1} \end{array}$$

LP optimum: $x_4 = x_5 = x_6 = x_7 = 1,\ x_1 = x_2 = x_3 = 0$

Challenge for the solver: To time-efficiently find valid and useful cuts

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Branch-and-Bound: An Example

Same knapsack problem instance:

$$\begin{aligned} \max \quad & 6x_1 + 4x_2 + 6x_3 + 7x_4 + 5x_5 + 9x_6 + 8x_7 \\ \text{s. t.} \quad & 5x_1 + 6x_2 + 8x_3 + 6x_4 + 4x_5 + 6x_6 + 5x_7 \leq 21 \\ & \boldsymbol{x} \in \{0, 1\} \end{aligned}$$

LP optimum: $x_1 = x_5 = x_6 = x_7 = 1$, $x_4 = 0.167$, $x_2 = x_3 = 0$

Rounding down x_4 to zero gives an integer solution of value $6 + 5 + 9 + 8 = 28$
$\Rightarrow$ Integer optimum is at least 28

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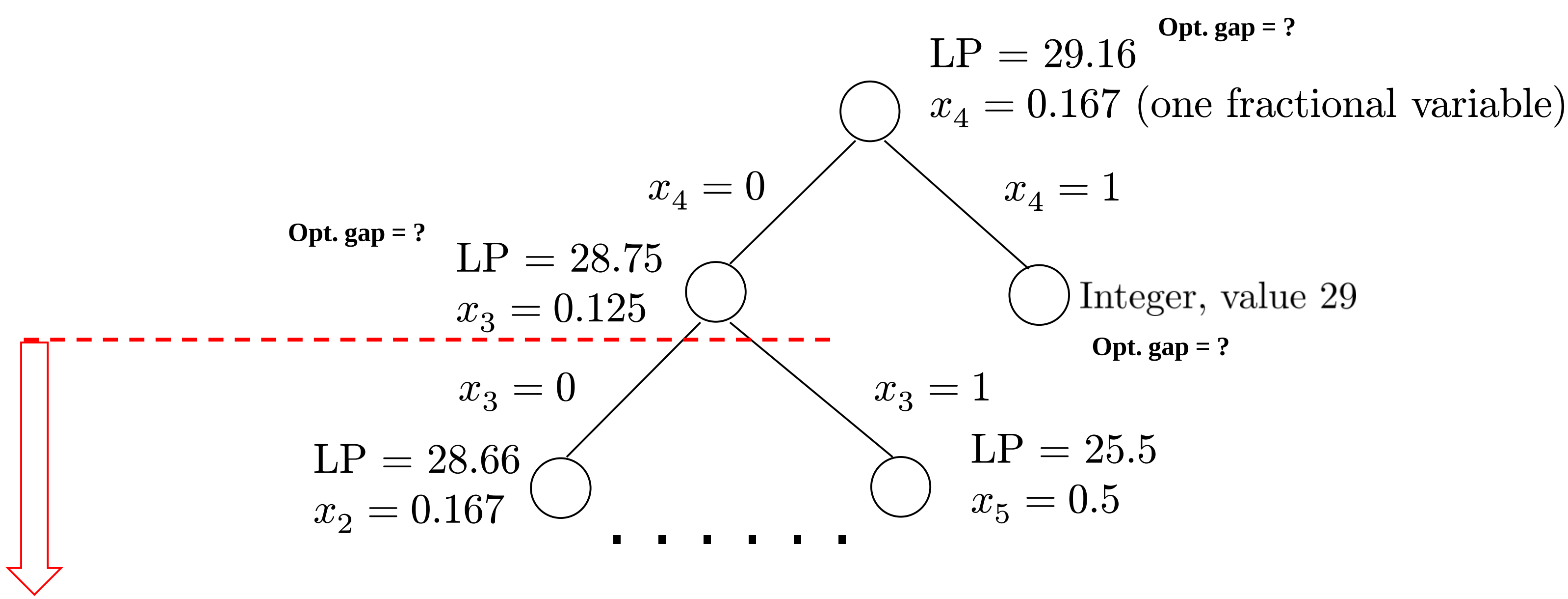
Branch-and-Bound: An Example (cont'd)

Branching generates a search tree



We can stop branching here if the integer solution of value 29 is known (why?)

Can we stop branching here because of an integer solution of value 28?

Optimization Solver

- Solver: software implementing methods for solving optimization models (here: integer programming models)
- Interface + optimization engine
- Many solvers: Gurobi, CPLEX, FICO Express, SCIP, MINTO, …
- Using Python to interact with solvers has become quite popular

Solver interface

- C/C++, Python (Gurobipy), Java, R, ...
- Command line
- AMPL, GAMS...

Gurobi optimization engine

- C/C++/C#, Python, Java, …
- Command line
- CPLEX OPL, AMPL, GAMS...

CPLEX optimization engine

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Sample of Solver Log

```
                 Nodes                                         Cuts/
    Node   Left      Objective   IInf   Best Integer      Best Bound      ItCnt        Gap

       0      0    227006.7258     3                   227006.7258      2417
*      0+     0                         232258.1349    227006.7258                 2.26%
*      0+     0                         228124.0562    227006.7258                 0.49%
       0      0    227020.2075     2    228124.0562       Cuts: 34      2475       0.48%
       0      0    227020.4963     1    228124.0562       Cuts: 31      2500       0.48%
       0      0    227020.5204     1    228124.0562       Cuts: 11      2505       0.48%
       0      2    227020.5204     1    228124.0562    227020.5204      2505       0.48%
Elapsed time = 2.02 sec. (969.11 ticks, tree = 0.01 MB)
*     12+     5                         227702.7631    227025.7507                 0.30%
      17      7    227161.7492     2    227702.7631    227025.7507      2710       0.30%
      78     50         cutoff          227702.7631    227034.4173      6190       0.29%
     148     83    227244.3466     1    227702.7631    227034.4173      9186       0.29%
     206    100         cutoff          227702.7631    227094.1950     11789       0.27%
     281    135    227586.9086     1    227702.7631    227179.3228     14822       0.23%
     349    166    227409.3745     1    227702.7631    227199.4808     17941       0.22%
     411    184    227634.5232     1    227702.7631    227255.7513     19929       0.20%
     467    195    227560.6384     2    227702.7631    227259.8459     21664       0.19%
     534    223    227494.7655     1    227702.7631    227269.0224     24768       0.19%
     789    260         cutoff          227702.7631    227336.1536     36761       0.16%
Elapsed time = 6.20 sec. (4102.59 ticks, tree = 2.69 MB)
    1028    250    227652.5107     1    227702.7631    227408.6859     42317       0.13%
    1349    121    227605.9339     1    227702.7631    227548.6905     53578       0.07%

Cover cuts applied:  14
Implied bound cuts applied:  291
Flow cuts applied:  4
Mixed integer rounding cuts applied:  17
Lift and project cuts applied:  1
```

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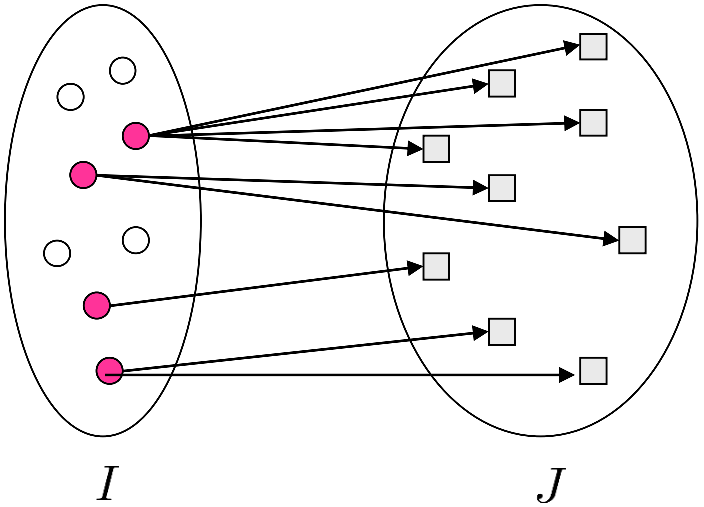
Uncapacitated Facility Location

Can we reduce the model size?

$$\begin{aligned} \min \quad & \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \\ \text{s. t.} \quad & \sum_{i \in I} x_{ij} = 1, \forall j \in J \\ & x_{ij} \leq y_i, \forall i \in I, \forall j \in J \\ & \boldsymbol{x}, \boldsymbol{y} \in \{0,1\} \end{aligned}$$



$$\begin{aligned} \min \quad & \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \\ \text{s. t.} \quad & \sum_{i \in I} x_{ij} = 1, \forall j \in J \\ & \sum_{j \in J} x_{ij} \leq |J| y_i, \forall i \in I \\ & \boldsymbol{x}, \boldsymbol{y} \in \{0,1\} \end{aligned}$$

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Uncapacitated Facility Location (cont'd)

Facilities	Customers	Aggregated Model (seconds)	Disaggregate Model (seconds)
20	200	1.19	0.39
30	300	5.55	1.6
50	500	69.67	19.42
70	700	2362.67	764.30

```
AMPL Version 20060626 (Linux 2.6.9-5.EL)
. . .
Node log . . .
Best integer =   1.433808e+05  Node =    0  Best node =
1.687869e+04
Best integer =   2.293498e+04  Node =    0  Best node =
1.687869e+04
Heuristic still looking.
Heuristic still looking.
Heuristic complete.
Best integer =   2.276127e+04  Node =  828  Best node =
1.831542e+04
Best integer =   2.267411e+04  Node = 1000 Best node =
1.838685e+04
Implied bound cuts applied:  2292
Flow cuts applied:  18
 . . .
Times (seconds):
Solve =  2362.67
CPLEX 10.1.0: optimal integer solution within mipgap or
absmipgap; objective 22674.11
1048766 MIP simplex iterations
16865 branch-and-bound nodes
```

```
AMPL Version 20060626 (Linux 2.6.9-5.EL)
. . .
Node log . . .
Best integer =   4.704366e+04  Node =    0  Best node =
2.203318e+04
Best integer =   2.295632e+04  Node =    0  Best node =
2.203318e+04
Heuristic still looking.
Best integer =   2.267411e+04  Node =    0  Best node =
2.203529e+04
Heuristic complete.

Gomory fractional cuts applied:  3
Using devex.
Times (seconds):
Solve =  764.304
CPLEX 10.1.0: optimal integer solution; objective 22674.11
157501 MIP simplex iterations
36 branch-and-bound nodes
```

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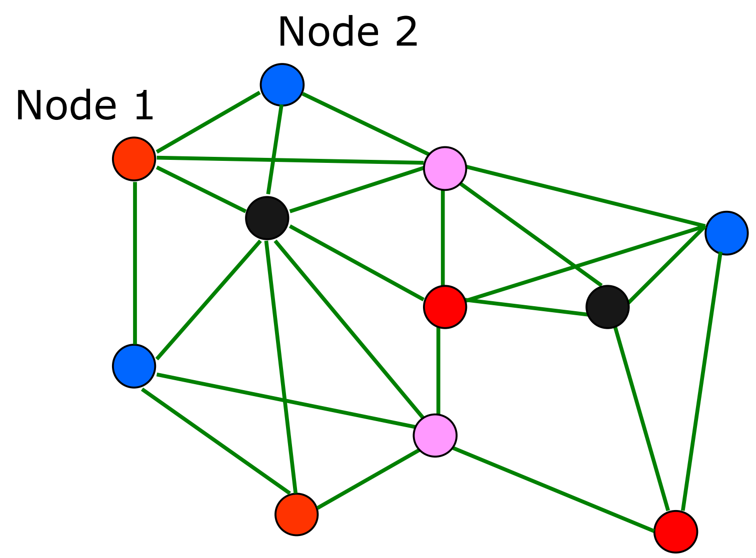
Coloring

Define color set C

$$
\begin{aligned}
\min \quad & y_{red} + y_{blue} + \dots \\
\text{s. t.} \quad & x_{1,red} + x_{1,blue} + \dots = 1 \\
& x_{2,red} + x_{2,blue} + \dots = 1 \\
& \dots \\
& x_{1,red} \leq y_{red} \\
& x_{1,blue} \leq y_{blue} \\
& \dots \\
& x_{1,red} + x_{2,red} \leq 1 \\
& x_{1,blue} + x_{2,blue} \leq 1 \\
& \dots \\
& \boldsymbol{x} \in \{0,1\}, \boldsymbol{y} \in \{0,1\}
\end{aligned}
$$

- $x_{ic} = 1$ if node i has color c
- $y_c = 1$ if color c is used (by some node)



- Symmetry: Solution search becomes inefficient
- Alternative model of set-covering type

Is this a good formulation? Other formulations?

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Coloring (cont'd)

Sample results for an extension of graph coloring:

Nodes	Model I (previous slide)		Model II (not shown)	
	Best Solution	Time	Best Solution	Time
10	10	0.1s	10	0.1s
20	16	1s	16	3s
30	21	≥10h	21	7s
40	15	≥10h	15	32s
50	28	≥10h	23	1m19s
60	31	≥10h	26	4m31s

It may matter (a lot) which mathematical model you use

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Final Remarks

- Integer linear programming (ILP) provides one tool (and a powerful one in many cases) for combinatorial optimization
- An ILP model: Linear functions of integer/binary variables for stating the objective function and constraints
- How to solve it: linear programming relaxation, cutting planes, and branch-and-bound, implemented in modern solvers
- Modeling (how to express your problem as ILP) may be very crucial for solution efficiency
- Recent trends include interaction of ILP and machine learning (ML)
  - ML for ILP: How to branch? What cutting planes to use?
  - ILP for ML: Robustness check of trained neural networks

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Appendix: Introduction to Modeling with AMPL

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Modeling Language: Separation between Model and Data

knapsack.mod

```
# Number of items
param NumItem >0;

# Set of items
set ITEMS := 1..NumItem;  # Creates set {1, ..., NumItem}

# Other parameters
param Limit >0;
param Value  {ITEMS} >=0;
param Weight {ITEMS} >=0;

# Variable definition
var x {ITEMS} binary;

# Objective function
maximize TotalValue: sum {j in ITEMS} Value[j] * x[j];

# Weight limit constraint
subject to WeightLimit:
sum {j in ITEMS} Weight[j] * x[j] <= Limit;
```

knapsack6.dat

```
param NumItem := 6;

param Limit := 14;

param: Value Weight :=
1        12      2
2        10      1
3        25      5
4        11    2.5
5        30      4
6        24      3;
```

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Command Script: An Example

```
# Reset AMPL
reset;

# Load AMPL model and data
model knapsack.mod;
data knapsack500.dat;

# Set solver parameters
option relax_integrality 0;
option cplex_options 'mipdisplay=2 integrality=1e-9 optimality=1e-9 timelimit=3600';

# Solve the problem
solve;

# Display optimum and solution
display TotalValue;
display {j in ITEMS: x[j]=1} x[j];

quit;
```

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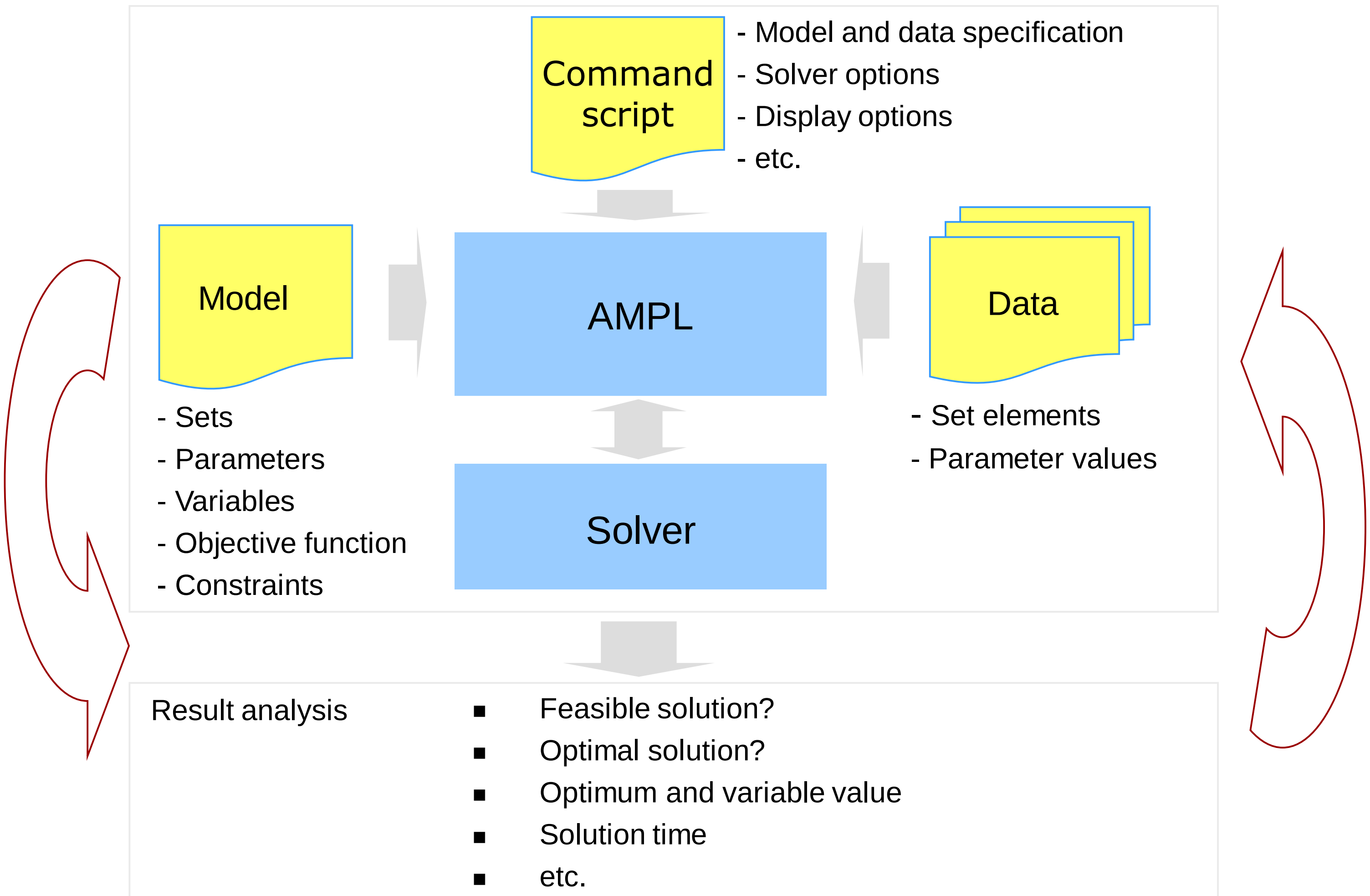
AMPL+ Solver



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AMPL Sets

- Simple sets (numbers or symbols)

```
set ITEMS := 1,2,3,4,5,6;
```

```
set ITEMS := 1..6;
```

```
set DAYS := Mon, Tues, Wed, Thurs, Fri, Sat, Sun;
```

- Indexed collection of sets

```
# Declaration of base stations, test points, and coverage relation
set BASESTATIONS;
set TESTPOINTS;
set COVERAGE {TESTPOINTS} within BASESTATIONS;
```

```
# Numerical values in a data file
set BASESTATIONS := 1..100;
set TESTPOINTS := 1..10000;
set COVERAGE[1] := 1 3 10 15;
set COVERAGE[2] := 2 3 7 8 11 19 25;
. . .
```

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AMPL Basics: Parameters

- Scalar parameter and parameters for set elements

```
param Limit;
```

```
param Capacity {Links};
```

- Bounds and default value

```
param Limit >0;
```

```
param MaxCapcity;
param Capacity {LINKS} >=0, <=MaxCapacity;
```

```
param Cost {BASESTATIONS} >=0 default 1000;
param Traffic {TESTPOINTS} >=0 default 0;
```

- Symbolic parameters

```
set DAYS;
param FirstDay symbolic in DAYS;
param LastDay symbolic in DAYS;
```

```
# Values in a data file
set DAYS := Mon, Tues, Wed, Thurs, Fri, Sat, Sun;
param FirstDay := Mon;
param LastDay := Sun;
```

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AMPL Basics: Variables

- Similar to declaration of numerical parameters
- May have value and/or type restrictions

```
var x {ITEMS} binary;
```

```
var production {DAYS} >=0, integer;
```

```
var flow {(i,j) in LINKS} >=0, <=Capcity[i,j];
```

```
var location {BASESTATIONS} binary;
```

```
var serve {TESTPOINTS, BASESTATIONS} binary;

# A more efficient declaration using set COVERAGE
var serve {j in TESTPOINTS, i in COVERAGE[j]} binary;
```

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AMPL Basics: Objective Function and Constraints

- Integer linear programming: The objective is a linear expression of the variables

```
maximize TotalValue: sum {j in ITEMS} Value[j] * x[j];
```

```
minimize TotalCost: sum {i in BASESTATIONS} Cost[i] * location[i];
```

- Single constraint

```
subject to WeightLimit:
sum {j in ITEMS} Weight[j] * x[j] <= Limit;
```

- Indexed collections of constraints (with condition)

```
subject to ServiceCoverage {j in TESTPOINTS: traffic[j]>0}:
sum {i in COVERAGE[j]} serve[j,i] >= 1;
```

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AMPL Basics: A Complete Model for Set Covering

$$\min \quad \sum_{i\in I} c_i x_i$$

$$\text{s. t.} \quad \sum_{i\in I_j} x_i \geq 1, \forall j \in J$$

$$\boldsymbol{x} \in \{0,1\}$$

```
set CENTERS;

set POINTS;

set COVERAGE {POINTS} within CENTERS;

param Cost {CENTERS} >=0;

var x {CENTERS} binary;

minimize TotalCost: sum {i in CENTERS} Cost[i] * x[i];

subject to Covering {j in POINTS}:
sum {i in COVERAGE[j]} x[i] >= 1;
```